Brand Reallocation, Concentration, and Growth

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Abstract

This paper studies the macroeconomic implications of firm-branding activities. We show empirically that firms build market share by creating new brands, developing their existing brands, and buying established brands from other firms. Sales and prices of the underlying branded products tend to rise when a large firm acquires a brand from a small firm. To interpret these findings and quantify the implications, we introduce an endogenous growth model where brand creation, maturity, and reallocation determine both market concentration and economic growth. On net, brand reallocation improves efficiency in the quantified model, even as it increases concentration by over 30%; blocking brand reallocation would reduce welfare by 2%. A tax on brand reallocation alleviates pricing distortions from concentration but nevertheless reduces efficiency by slowing growth. In contrast, a subsidy to brand or firm entry can alleviate pricing distortions and raise growth. In markets with fast maturing brands, subsidies to entry become more effective and blocking brand reallocation becomes more costly. Broadly, our framework finds that effective industrial policies require attention to brand maturity, heterogeneity, and fit with the production and distribution capabilities of the parent firm.

Key Words: Endogenous Growth, Firm Dynamics, Productivity, Market Concentration, Product Innovation, Reallocation, Mergers & Acquisitions, Brands, Trademarks, Intangible Assets

JEL Code: O31, O32, O34, O41, D22, D43, L11, L13, L22

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1 Introduction

Brands are an essential intangible asset for firms. According to recent estimations of brand value, the top 100 brands in the US economy were worth \$4.14 Trillion in 2021, and the relative value of brands to traditional capital has been growing over time (Bronnenberg et al., 2022).[1] Brands allow firms to differentiate their products from competitors, and naturally affect a firm's pricing power on its branded products. Firms build brand capital by introducing new brands, growing their existing brands, and the acquisition of brands from other firms. Brands, like technology, drive both economic growth, through new product creation, and market concentration, through firms amassing customer capital. Despite rising interest in product market concentration and intangible assets, the macroeconomic implications of branding activities are less understood. This motivates the following research questions: how do branding activities, such as brand creation, maturity, and reallocation, affect market concentration and economic growth? How do they affect the efficiency of the aggregate economy?

To answer these questions, we build a new dataset that merges intellectual property data from the US Patent and Trademark Office (USPTO) with price and quantity retail scanner data from RMS Nielsen. The dataset provides information on products, brands, trademarks, and owning firms. We define the first three concepts in turn. A *product* is a consumer good in a given product group code as defined by RMS Nielsen.[2] A *brand* is an image or symbol that enhances the value of a product to the consumer beyond the product's functionality. A *trademark* is the intellectual property that grants the parent firm exclusive use of its brand. The USPTO defines a trademark as "any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's how customers recognize you in the marketplace and distinguish you from your competitors." In our empirical analysis, we focus on the sales and prices of a *branded product* within a product group.[3] With our new data, we document the following three facts regarding firms' branding activities:

Fact 1 *Large firms have significantly more brands and market share than small firms and build their market share through brand acquisition more than brand creation.*

Fact 2 *Brands build sales over time, and better and more mature brands are more likely to be reallocated across firms.*

Fact 3 *When a brand is reallocated across firms, the sales and prices of the brand both increase.*

The patterns in the data are striking. Both the market share distribution and brand holdings distribution are skewed. The largest firm in a given product market is over 1000-times larger than the median firm in

[1] This \$4.14 Trillion represents 0.47 of the size of the total value of Property, Plant, and Equipment at the same firms.

[2] RMS Nielsen uses product group codes to allocate similar goods into 116 categories, such as "Cereal" and "Beer".

[3] Our analysis identifies information at the brand $\times$ group level as defined by RMS Nielsen, collapsing products into their common brand umbrella, which we refer to as a brand or branded product, as discussed in Section 2.

market share and has 27-times more brands than the median firm, which has only 1 brand. The market share distribution at the brand level is highly skewed as well. The largest brand in a given product market is over 1000-times the size of the median brand. Branding activity is dynamic, both in terms of creation and reallocation. On average, more brands are created (10% per year) than reallocated (1.9% per year). However, newly created brands are smaller than reallocated brands due to brand maturity and heterogeneity. As a result, brand reallocation is more significant than brand creation in terms of market shares (2.2% v. 1.1% per year). When brands are reallocated from small to large firms, sales revenue at the brand-level (in this paper referred to as brand sales) increases as do average log prices.

To connect these facts and answer our questions on the macroeconomic implications of branding, we introduce an endogenous growth model with brand creation, maturity, and reallocation, allowing for both welfare gains through love-of-variety and welfare losses due to markups. In our model, consumers spend on imperfectly substitutable products distinguished by their brand. This imperfect substitution among branded products mirrors the role of brands in reality: all firms brand their products to some degree, and firms apply early in their life cycle for trademarks to protect their brand capital. In our model, each product group category has a multi-product leader and endogenous measure of single-product fringe firms, mirroring the disproportionate size of large firms in the market. The multi-product leader charges a higher markup than fringe firms because, as it holds many brands, it internalizes its impact on group-level prices.

In the model, firms build their market shares through three types of activities. They introduce new brands through *brand creation*, exploit the existing *brand maturity* which occurs exogenously (e.g., consumer word-of-mouth), and search to acquire existing brands from other firms through brand acquisition or *brand reallocation*. While searching for brands to acquire, firms seek to maximize their bilateral gains from trade. In the model, the gains from trade originate from two sources. First, the buying firm can be more efficient in operating a brand, which we refer to as *efficient reallocation*. Second, the buying firm may use the brand to exert a higher markup, which we refer to as *strategic reallocation*.

The efficiency of the decentralized equilibrium depends on both the number of branded products created and the level and distribution of markups across firms. Firms do not fully internalize their activities on either the consumers' love-of-variety or the distortions from markups. These externalities imply that the economy faces an *efficiency-markup tradeoff*. With efficient reallocation, the buying firm is better equipped to produce and distribute the brand due to firm-specific advantage or firm-brand fit. In this case, the interests of the firms and the aggregate economy align. On the other hand, strategic reallocation occurs because the buying firm can exert market power due to limited consumer substitution across products. Here, the firm's interest in consolidation may not align with overall economic efficiency. While we consider these two types of transactions as benchmarks, many brand exchanges can (and do) exhibit evidence of both efficient and strategic components.

Whether a brand ownership reallocation is efficient or strategic generates different predictions for prices and sales. In efficient exchanges, sales should increase while prices decrease. In strategic exchanges, prices rise and sales stay flat or decline. If exchanges are a mix of strategic and efficient, we may see a mix of these outcomes. In the data, we find evidence of both types of exchanges. In an average exchange from a small to large firm, sales go up by 47% but prices also increase by 6%. This mix of strategic and productive aspects of reallocation have ambiguous effects on welfare, which provide an important motivation for quantifying the model.

To better match the reality of consumer product markets, we augment the model with the following features: (1) we assume brands are born heterogeneous and experience life cycles (as noted in our empirical facts and Argente et al., 2020a); (2) we assume firms differ both in their fixed characteristics and their match quality with brands, which leads to the reallocation of brands in both directions among large and small firms; (3) we introduce a tax/subsidy on both innovation and reallocation activities to evaluate the policy implications. With these additions, the growth rate, concentration, and policy implications in our model are driven by both firm dynamics and brand dynamics, as well as their joint movement.

We use the estimated model to decompose the sources of concentration and growth and study policy counterfactuals. First, we find that reallocation explains around 30% of the market concentration in the typical product group. Even though reallocation contributes significantly to concentration, shutting down reallocation leads to welfare losses of 1.93%; a 10% tax on reallocation reduces welfare by 0.43%. This welfare loss occurs due to both the immediate reallocation effect and declining entry, as potential entrants do not have the incentive to create and sell their brands. Subsidizing entry is a more efficient policy, as it increases growth and reduces concentration; a 10% subsidy leads to a 5.84% increase in welfare in the balanced growth equilibrium.

Brand maturity and reallocation have a critical interaction. In markets with fast maturity, e.g., firms can build brand capital quickly, brand reallocation is less costly and induces more entry. Even strategic exchanges may be efficient, and shutting down reallocation is very costly. In markets with slower maturity, we find brand reallocation can be inefficient since the strategic effect dominates. Our findings suggest that optimal policy should incorporate group-level fundamentals. One policy that is effective for cereal may not be appropriate in apparel.

The remainder of this section reviews the literature, while the rest of the paper is structured as follows. Section 2 introduces the USPTO Trademark Dataset and RMS Nielsen Scanner Data and discusses the merge between the two datasets. Section 3 documents the key empirical facts that frame our investigation at the firm and brand level. Section 4 introduces a model of brand creation, maturity, and reallocation with variable firm productivity and variable markups. Section 5 estimates the model. Section 6 uses the quantified model to understand the contribution of specific margins and perform policy counterfactuals. Section 7 concludes.

Related Literature

This paper builds on and contributes to several literatures: the study of firm dynamics and product dynamics; the study of concentration, innovation, and firm profitability; the macroeconomics of M&A and technology transfers; and the study of brands and branding.

Firm performance is inextricably linked to its branded products. Hottman et al. (2016) study multi-product firms and find the scope of products explains a large share of sales variations across firms. Argente et al. (2018, 2020a) explore how product creation and destruction are pervasive in product markets. Further, Argente et al. (2021) and Einav et al. (2021) document that the expansion of product sales is primarily due to the expansion of customer base. We connect these important empirical insights to a firm's decision when they hold many products. Atkeson and Burstein (2008) introduce oligopolistic competition into a model with large multi-product firms, which is a building block of our model. Our paper links these papers by linking the product or brand life cycle to product innovation and reallocation in an environment where firms have variable markups as in Atkeson and Burstein (2008) and product substitution shapes firms' decisions, productivity, and market power (e.g., as noted by Syverson, 2004a,b; Melitz and Ottaviano, 2008).

By incorporating product innovation and reallocation jointly, we speak to a literature where product innovation is at the center of economic growth dating back to Romer (1990) and Grossman and Helpman (1991). This link has also been documented empirically, as product creation plays an essential role in both growth and the gains from trade, as noted by Bils and Klenow (2001), Broda and Weinstein (2006), Argente et al. (2018), and Jaravel (2018). Yet, this literature has not allocated attention to the role of brands in the macroeconomy, even though innovations and new products are always linked to brands, and thus branding has significant firm value and growth implications.

This paper addresses market concentration and innovation both theoretically and empirically. Recent work has focused on rising markups (e.g., De Loecker and Eeckhout, 2018; De Loecker et al., 2020), rising concentration (Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018), and the rise of superstar firms (Autor et al., 2020). To connect these discussions to growth, our model builds on the long literature of endogenous growth through creative destruction (Aghion and Howitt, 1992, Aghion et al., 2001, Akcigit and Kerr, 2018, Peters, 2020, and Liu et al., 2022), augmenting this with literature on entry and firm development (Jovanovic, 1982; Hopenhayn, 1992). Some papers in this tradition focus on the links between factor or labor reallocation and growth (Acemoglu et al., 2018; Garcia-Macia et al., 2019), while we focus on intangible capital. Jones and Williams (1998, 2000) study how markups and innovation interact to determine over- or underinvestment in R&D. Through addressing how firms are motivated to create new innovations and products by incorporating the markup value and restricting output, they find firms underinvest in research. This underinvestment can be alleviated through the subsidy of new varieties.

We explore a similar mechanism in this paper. Edmond et al. (2015) also focus on the markup channel, as large firms can leverage their large market share to charge high markups. Akcigit and Ates (2019, 2021) focus on the knowledge diffusion gaps between leaders and followers driving rising concentration and falling business dynamism. De Ridder (2019) focuses on intangible capital as a barrier to firm entry. Cavenaile and Roldan-Blanco (2021) study the interaction between firms' advertising activities and growth, while Greenwood et al. (2021) focus on the macroeconomic effects of targeting advertisement.

To speak to the joint determinants of innovation and reallocation, our paper combines methodologies from the labor search and matching literature to topics in economic growth. Theoretically, we build on Menzio and Shi (2011) by embedding a directed search model into a growth framework. This differs from Lentz and Mortensen (2008), who embed random search into a growth framework to study reallocation and innovation. Empirically, we build on frameworks that study reallocation, mostly in the labor context, relating to work dating back to Davis and Haltiwanger (1992) and Davis et al. (1996). We note that similar measures can be used with intangible assets.

The reallocation of intangible assets connects to questions on the aggregate implications of mergers and acquisitions (M&A) and patent acquisitions. David (2020) studies the aggregate implications of M&A through the lens of a random search model and finds M&A increases overall efficiency. Akcigit et al. (2016) study intellectual property misallocation and the market for patents, and find that this secondary market increases efficiency. Eaton and Kortum (1996) and Shi and Hopenhayn (2017) study how the appropriability of innovation, the ability to license or sell intellectual property, induces upstream incentives to innovate. Abrams et al. (2019) find evidence that intermediaries in intellectual property transfers exhibit both positive and negative effects on downstream innovation, while Cunningham et al. (2021) find that "killer acquisitions", where incumbents acquire products to kill them, have an important role in pharmaceuticals. In our model, similar mechanisms are present that we discuss in detail in Section 4. Two recent papers discuss the role of antitrust policies on growth, from the perspective of technological innovation (Cavenaile et al., 2021 and Fons-Rosen et al., 2021). Our theoretical framework relates to these papers in integrating the dynamic effects of transactions, but differs in (1) the focus on brand capital (2) in an environment with endogenous variable markups. In our model, market concentration and growth are endogenous and impact household welfare, allowing us to discuss the benefits and costs of various antitrust and innovation policies.

Lastly, we bring key insights from the literature on brands and branding to the macroeconomic debates on concentration and growth. Brands have long been known to be an important source of firm values (e.g., Braithwaite, 1928 on advertising and Brown, 1953 on trademarks). Bain (1956) noted that "(t)he advantage to established sellers accruing from buyer preferences for their products as opposed to potential-entrant products is on the average larger and more frequent in occurrence at large values than any other barrier to entry." Theoretically, brands can generate persistent profits in markets with imperfect information (Shapiro, 1983) and have value in exchange across firms (Tadelis, 1999). The power of branding has been detailed empirically as consumer brand preferences are quite persistent (e.g., in Bronnenberg et al., 2009, 2012) and thus provide firms significant value. Dinlersoz and Yorukoglu (2012) develop a model linking customer knowledge of products to firm growth. Gourio and Rudanko (2014) note how customer capital is a relevant state variable for firms and they bring this customer capital into firms' dynamic behavior. Heath and Mace (2019) show empirically how competition over customer capital generates strategic behavior in the market for trademarks, and is consistent with the significant degree of activity in the market (noted by Kost et al., 2019). This current paper builds on these papers in two respects. First, we link brand capital to the product market shares of firms and the aggregate economy. Second, we study how brands can be reallocated across firms, which makes the distribution of brand capital across the economy a key state variable of interest. We first turn to the underlying data regarding brands to establish the links between brands and firms.

2 Data

This section links the products, brands, and firms in the two datasets and provides summary statistics on the datasets and the merge. We start by directing our conceptual focus to the basic unit of analysis in this paper, the *branded product*. A branded product is the set of products merged to a trademark owned by a firm within a specific product group. For example, the branded product "Cheerios" has a brand name, Cheerios, is in a product group "Cereal", and is owned by General Mills. As a result, we will not distinguish various products that have the same brand and group but different features identified by a unique UPC code (e.g., Cheerios family size versus Cheerios regular size). Unless we specify *UPC* products, our reference to a product will reference the set of products under the brand, which is our unit of interest. Our merge focuses on building this consistent definition by linking intellectual property with price, quantity, and at the brand and group level.

We first motivate our empirical framework and then turn to the two datasets that serve as the bedrock for our empirical analysis. Our framework splits a branded product's sales outcome into three components. The sales of branded product i in firm j at time t (sales c_{ijt}) is a function of the owning firm, the brand, and a match-specific component (the firm-brand fit) as follows,

$$c_{ijt}(branded\ product_{ijt}) = \alpha(firm_{jt}) + \beta(brand_{it}) + \gamma(brand_{it} \times firm_{jt}).$$

The most appropriate dataset to understand these forces would be at the brand and firm level and would provide detail on brand history, including the prices, sales, and age of each brand, and firm-specific features, such as firm's sales revenue and brand holdings. This paper applies USPTO trademarks and

RMS Nielsen scanner data to track the creation, distribution, prices, and quantities of branded products.

The trademark data provide the history of each brand and parent firm in terms of registrations, cancellations, and transactions or reallocation. To focus on the dynamics of prices and quantities, we connect these firm- and brand- level data to specific information on product prices and quantities sold by stores in RMS Nielsen Scanner Data. To separately identify the effects of brands and firms, we rely on transactions of brands across firms. The following two sections discuss these two datasets central to our empirical analysis.

2.1 USPTO Trademark Data

USPTO Trademark data provide a unique and comprehensive insight into the distribution and history of brands across firms. Trademarks are a central and dynamic arena of the economy, as firms register for trademarks whenever they want their brand to be legally protected. Trademarks are common, and more firms participate in trademarking than patenting.

When firms create new brands, they apply to the USPTO to protect the consumer appeal associated with the brand. Further, when firms buy the rights to brand ownership from other firms, the trademark is reassigned across firms.

To register for a trademark, a firm must undergo the following process. First, an individual or firm who applies must pay a fee that ranges from $225 to $400. Within three months of filing, an examining attorney checks for compliance, and if the application is approved, it "publishes for opposition." A 30-day period follows, during which third parties affected by the trademark registration can step forward to file an "Opposition Proceeding" to stop the registration. This process is again evaluated by an examiner. If it clears this process, the trademark is registered.

The owner of a registered trademark has exclusive rights to use the mark within the sphere of activity designated by the legal process. Such rights include indefinite renewal conditional on continued use and the right to exchange a trademark. Dinlersoz et al. (2018) and Kost et al. (2019) discuss the institutional aspects of trademarks in greater detail. Further, Appendix A presents some examples of firms with multiple brands and the firms' motivations for brand reallocation from press releases. Here, we turn to summary statistics on the number of trademarks and their distribution across firms. Table 1 provides details on the number of firms and trademarks and the distribution of trademarks across firms.

We focus on two features of the data from Table 1. First, the number of unique brand transactions is almost as large as the number of registered brands, indicating a constant flow of ownership across firms. While we focus on reassignments and mergers, there are multiple types of transactions, which we discuss in detail in Appendix B.4. Another striking feature of the data is the skewness of firm size. The 99th percentile firm is over 80-times larger than the median firm in terms of the stock of trademarks. We note

Table 1: Summary Statistics on Trademarks from USPTO

Data Object	Count
# unique firms	1.35M
# unique registrations	5.36M
# unique transactions by bundle	915,076
# unique transactions by ID	4.46M
# unique cancels	2.12M
99th percentile firm size	83
75th percentile firm size	5
Median firm size	1
Mean firm size	5

Notes: Variables taken over entire sample of data, with size variables taken in 2010. Firm size is defined as the number of trademarks within a firm. Source: USPTO Trademark Data.

even more stark patterns in terms of sales, and this recurrent pattern of concentration is a central aspect of our data. Without detailed price and sales-level data, the efficiency implications of this concentrated ownership are unclear. Linking brands to prices and sales is the next step in uncovering these forces.

2.2 RMS Nielsen Scanner Data

Detailed brand- and product-level data are essential to the empirical and quantitative analysis of this paper. We apply store-product-level data that come from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space from years 2006–2018. Although we apply historical use of trademark analysis to understand the age and evolution of brands, 2006–2018 is our primary focus for observing market shares.

We have more than 100 billion observations at the UPC product $\times$ store $\times$ time level. A UPC product in Nielsen (distinct from the branded product in our definitions) is defined by a UPC identifier, 12 digits that are uniquely assigned to each specific good. The store is defined at the local level, with over 40,000 total; time is defined weekly but we collapse to annual data for our analysis. Total sales are approximately $300 billion per year, covering around half of consumption in the grocery, drug, and merchandise stores (Argente et al., 2020a), which itself covers approximately 8% of total GDP consumption. We apply a dataset from GS1 US, the official source for UPCs, to link parent firms to brands through each UPC.

The UPC barcodes provide a unique identifier for each product. Changes in any attribute of a good corresponds to a new barcode. Barcodes are widespread and thus cover a large amount of the CPG industry. However, the unique identifying feature of the barcodes may not be as relevant for our analysis.

For instance, the parent trademark associated with "Coca-Cola 20oz" is the same as "Coca-Cola 12oz".

One important departure from the literature in this paper is focusing on *brands* or *branded products*, defined by brand names and their underlying products in a given group, rather than *UPC products*, that is, UPC codes (with specific firm ID and characteristics, e.g. 12oz). We discuss three reasons for focusing on brands rather than products. First, consumer goodwill tends to be brand- rather than UPC-specific. Coke 12oz relies on the same core branding as Coke 20oz. Thus, the brand is a more central indicator relevant to customer capital and firm valuations. Second, when firms exchange brand ownership, or the right to sell a specific brand, they tend to systematically transfer the full rights on the consumer goodwill, making the specific product differentiation within the brand less relevant. Third, our data enable identification at the brand level in both the Nielsen data and USPTO trademark data. Nielsen provides brand identifiers in addition to product identifiers. We collapse this information into brand $\times$ product group $\times$ year. After collapsing products to the brand level, we focus on *branded products* as the consumption good linked to a unique brand for the rest of this paper. We do not focus on store or geographical variation in this paper. While GS1 links to most parent companies, the USPTO trademark dataset complements GS1 to ensure the parent company is allocated to the correct brand and augments the data by delivering parent companies.

2.3 Combining the Datasets

To link brand age, brand exchange, and brand evolution, we employ a fuzzy merge to connect brand names in RMS Nielsen scanner data to USPTO trademark data. Whereas this merge is the first we know of that links USPTO *trademark* data to Nielsen scanner data, Argente et al. (2020b) link USPTO *patent* data to RMS Nielsen data. We follow a similar method but get greater coverage in our merge, likely due to the different nature of patents and trademarks. In particular, we are able to identify all products connected to their brand name as long as the trademarked brand name is similar enough to the brand name on the product in the store.

We start by normalizing names in each dataset at both the firm- and brand-level. For example, we want to capture heterogeneous naming at the firm (e.g., General Mill Holdings + General Mills Minnesota Op.) and connect it to the parent company. We then turn to the brands themselves. We employ a similar fuzzy match with brands. We start by linking observations at the firm $\times$ brand level, but for observations for which we directly observe the brand, we connect the brand independently and assign ownership through trademark data. We discuss the mechanics of the merging process in Appendix A.2.

Both USPTO and Nielsen scanner data contain a firm $\times$ brand observation of interest. The identification of firms and brands provides what is ideally a many-to-1 matching between Nielsen brands and trademark brands. However, we find often there are multiple trademarks associated with multiple brands.

For these matches, we focus on the most reliable name match. If the same brand has multiple matches, we take the "active" brand. For instance, if a brand is reassigned across firms, we assume this represents the focal brand. Once this match is complete, we have the core data concepts in our paper, the branded product (or brand $\times$ product group code), which is sold to the consumer and which we refer to as a brand holding within the firm. We leverage firm ownership coverage from both data sources. In general, we take the RMS Nielsen data unless the trademark indicates a *transfer* of ownership, which covers approximately 20% of the exchanges in our data.[4] The combined dataset delivers our indicator of brand ownership, age, sales, and prices.

We next turn to the quality of the match between brands in USPTO and RMS Nielsen. Table 2 provides information on the match between products and trademarks.

Table 2: Summary Statistics on Trademark–Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Brands	5.36M	1870–2020	1.9%
Firms	371,021	1870–2020	15%
Canceled Brands	2.12M	1970–2020	
Transactions	915,076	1970–2020	
RMS Nielsen Scanner Data			
Products $\times$ Group	1.64M	2006–2018	
Brands $\times$ Group	82,525	2006–2018	57%
Firms	23,232	2006–2018	54%
Brand $\times$ sales		2006–2018	82%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data

We stress a couple points from Table 2. First, when we merge brands with sales weights, we capture 82% of sales in the data. Without sales weights, we capture fewer brands. Some small firms may choose not to protect their intellectual property via legal means. Second, many trademarks are not associated with consumer packaged goods, so a smaller share of trademarks are merged. We also find that, not only are multiple brands associated with single firms, but also that multiple UPC products are connected to a single brand. On average, we observe 13 unique UPC products per branded product. From now on, we collapse this information into the branded product units to study brand and firm dynamics jointly.

[4]For aggregate activity measures applied in the quantitative section, we apply transaction data when the buyer and seller are different, but may have an unidentified buyer or seller in our data.

3 Empirical Analysis

This section focuses on the empirical observations that inform our model and quantitative analysis. We focus on three main margins. We focus first on the firm-level margin and discuss firm concentration and firm dynamics. We then turn to the brand-level margin, where we focus on brand heterogeneity and brand dynamics. We then turn to firm $\times$ brand analysis, focusing on brand-ownership transactions and the outcomes of reallocation events.

This section proceeds in three steps. Section 3.1 starts by focusing on firms. We start by documenting the degree of dominance of firms in product markets, illustrating the role of market leaders and their persistence. Second, we decompose the forces that contribute to firm-level market share. We focus on three core drivers of concentration: (i) brand creation and destruction, (ii) existing brand maturity and sales growth, and (iii) brand reallocation across firms.

Section 3.2 unpacks brands more directly, turning attention to the three core forces contributing to concentration — brand creation, maturity, and reallocation of ownership across firms. The brand life cycle exhibits striking patterns in the data. Older brands account for the largest share of sales. Brands exhibit a similar pattern with transactions, as older and larger brands are more likely to be transacted. Both transaction rates and sales revenue exhibit declines later in life.

Section 3.3 focuses on the rate and nature of brand reallocation across firms. We document evidence of efficiency gains from brand reallocation as sales revenue increases, as well as strategic gains, as prices increase. We discuss how these three forces interact, and their aggregate implications, in Section 5 and Section 6 respectively.

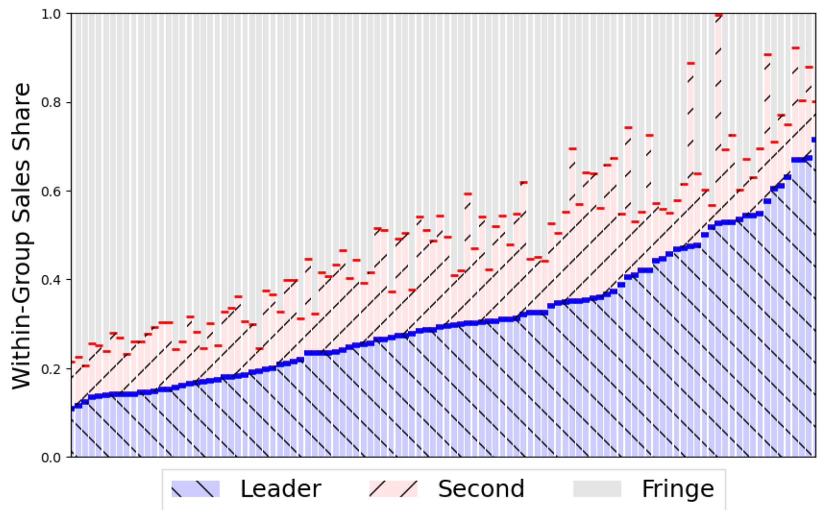
3.1 Firm-Level Analysis

All product sales accrue to firms. In this section we focus on the level, persistence, and sources of market shares. We first focus on overall concentration and then turn to the persistence of market leadership. We then analyze the dynamic elements driving concentration, decomposing the sources of concentration into brand entry/creation, brand maturity/growth, and brand reallocation.

Figure 1 maps out the sales share of the product leader, the second firm, and the remaining firms in the market. This split by product group contains 116 unique product-group categories (e.g., “Ice cream" or “Beer”). The average top firm share is 31.6% of the total market, while the top two firms hold 48.1% of the market. The top two firms account for about half the sales revenue in a given market. We also note the presence of a host of small firms (median share of 0.01% of the market), and in our framework, we think of these firms as “fringe”, since they hold few products and a small market share.

Leading firms are also quite persistent. Across all categories, the leading firm in one period has a 97% chance of being among the top two firms in the product group in the next period. Firm-level concentration

Figure 1: Sales Share of Leader, by Product Group



Note: *This figure shows the sales share by product group (ordered by % share of leader) in 2010.* Source: *RMS Kilts-Nielsen Data Center & GS1 firm-product merge.*

in product markets is persistent, yet it is not made up of single brands. On average, market leaders hold 27 unique brands within the product group they lead. Variation in concentration is closely connected to how firms develop and hold market shares with their brands.

Product market dominance does not happen in a day. Concentration is the outcome of long-run competition in the market shares of products across firms. The brand life cycle is intertwined with firm growth and decline through three core channels. First, and most noted within the innovation literature, is brand (or product) creation and destruction. Second, once brands are born, they mature (and decay) over time. Third, brand ownership is reallocated across firms.

The structure of our data allows us to characterize these three forces in detail. We set up three regressions where the coefficients add up to 1, and each coefficient is linked to the amount of variation of firm growth and decline the margin explains. We run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \quad (1)$$

Equation (1) focuses on three different margins for y_{it}. We substitute each of the three margins discussed above as y_{it} (y_{it}=creation, maturity, reallocation). Product maturity can refer to either increases and declines in sales over time. We present the results of the three separate regressions in Table 3.[5]

We stress two main takeaways from Table 3. At the firm level, variation from entry is much more significant for fringe firms (e.g., small firms) than for large firms (0.091 versus 0.033). Reallocation is

[5]We take firms that are leaders in their product group and collapse this information to the overall firm level.

Table 3: Sources of Firm-Level Sales Growth and Decline

			Leader	
	Innovation	Incumbency	Reallocation	Entry/Reallocation
Value	0.033***	0.84***	0.13***	**0.25**
	(0.315)	(0.000)	(0.000)	
Observations	383	383	383	
			Fringe	
	Entry	Incumbency	Reallocation	Entry/Reallocation
Value	0.091***	0.89***	0.021***	**4.33**
	(0.000)	(0.000)	(0.000)	
Observations	95,353	95,353	95,353	

p-values in parentheses, clustered at product-group level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Market share reallocation is measured across different firm types, following Equation (1). Source: RMS Nielsen.

relatively much more important for sales variation for large firms (about 6-times larger). For fringe firms and market leaders, variation from incumbent products drives the largest share of firm-level variation. Some variation from incumbent products may come directly from the life cycle, whereas other variation may be due to idiosyncratic shocks. We focus directly on the role of life cycle variation in Table B1 in Appendix B. We find that the fitted life cycle explains a most of the variation at the firm level, with approximately 20% left as a residual.

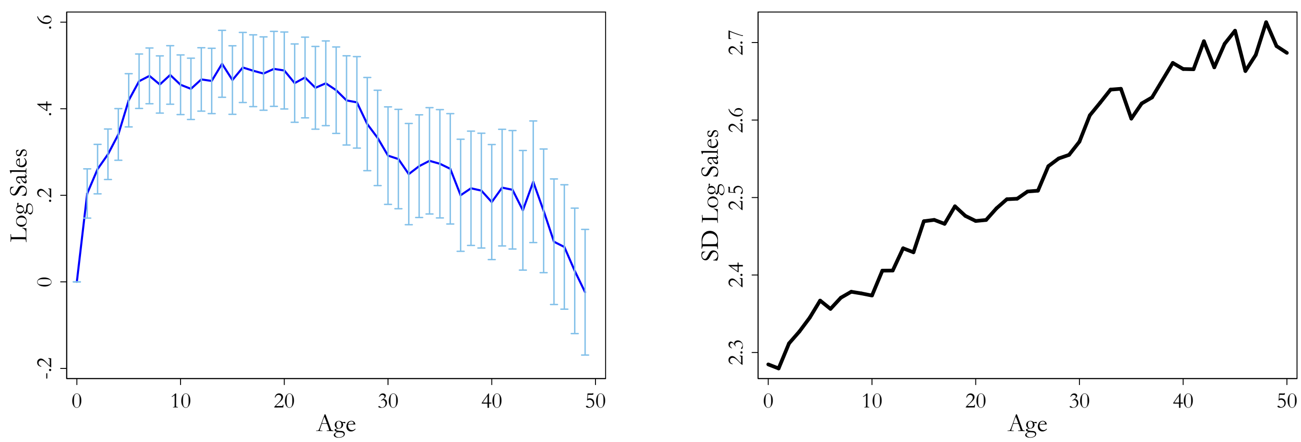
3.2 Brand-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016) and highly dynamic (Argente et al., 2020a), thus affecting the overall sales at the firm and product group level. All products are produced under a brand umbrella, and we focus on those jointly as the branded product. Our goal in this section is to focus on the interaction between brand heterogeneity, brand maturity, and brand reallocation. To study maturity, we study the life cycle of brands, where we leverage age data from trademarks registrations from the USPTO and sales data from RMS Nielsen. This study of the brand life cycle extends previous work, such as Fitzgerald et al. (2016) and Argente et al. (2018), who have studied the life cycle of products indexed by their product identifier. Figure 2 plots a regression that illustrates the nature of the product life cycle in sales, by plotting the coefficients from the following regression:

$$\log y_{it} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_i + \epsilon_{it}. \tag{2}$$

The regression in Equation (2) considers the sales of brand i at time t, $\log y_{it}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[6] The θ_i indicates a brand fixed-effect. Figure 2a plots the regressions by age coefficient β_a. The standard error bars indicate the 95-percent confidence interval for the point estimates for the regression with clustered standard errors at the brand level. Figure 2b tracks the within-age log sales dispersion across brands.

Figure 2: Brand-Level Dynamics



(a) Log Sales × Age Regression

(b) Log Sales Dispersion × Age

Notes: This figure plots a regression of log sales on age from Equation 2 (panel a), and the standard deviation of log sales within age (panel b). 95% confidence interval standard errors clustered at the brand-group level. Source: USPTO Trademark Data and RMS Nielsen

There are two main takeaways from Figure 2. First, brands exhibit an inverted-U pattern in sales over their life cycle. While consistent with the general pattern in the literature on the product life cycle, this brand life cycle is much longer and peaks far later than Argente et al. (2020a) and Argente et al. (2021). Second, the log sales dispersion is high initially and increasing in age. Brands are highly heterogeneous, and as they mature the sales dispersion varies more drastically. The life cycle itself drives not only level differences but within age heterogeneity.[7] In Appendix B, we explore brand heterogeneity and maturity in greater detail.

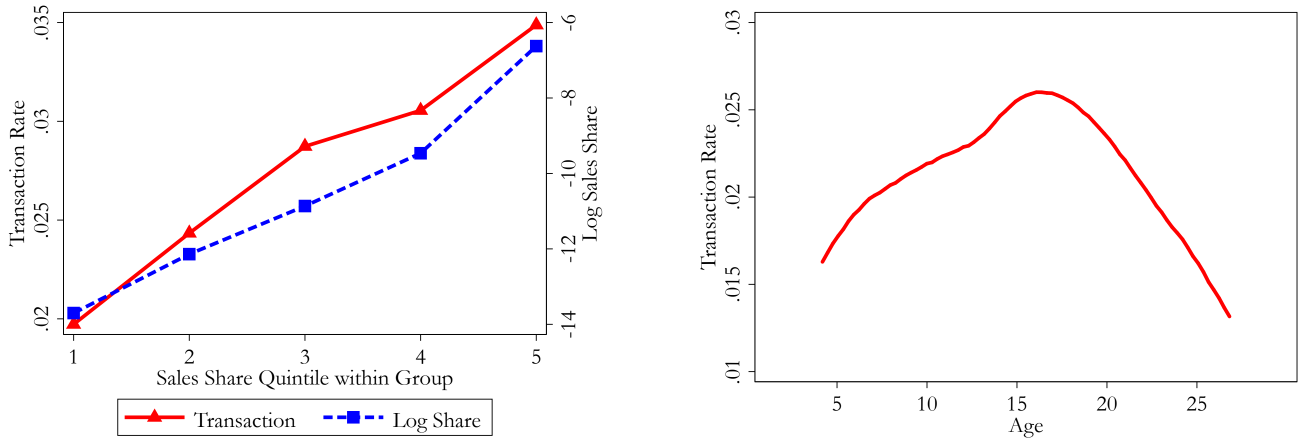
Not only do brand sales change over a brand's life cycle, but so do brand reallocation rates. Few brands are reallocated when very young, because they need time to build customer capital and exposure to other firms. Later, as brands decay (or find their best firm) they also experience a decline in the rate of reallocation.

Figure 3 focuses on the interaction between transaction rates, sales share, and age. Figure 3a focuses on the interaction between the sales share and transactions. Figure 3b leverages the history of USPTO trademark data to understand the interaction between brand age and transaction.

[6]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

[7]This is consistent with the theoretical predictions from brands learning their type, as noted in firm dynamics by Jovanovic (1982) and Hopenhayn (1992).

Figure 3: Transactions, Age, and Sales



(a) Transaction Rate by Quintile

(b) Transaction Rate by Age

Notes: Panel (a): Transaction rate by sales share. Panel (b): Transaction rate by age. Source: USPTO Trademark and RMS Nielsen.

Figure 3a splits the brands into quintiles (truncating brands with less than $1000 in sales in a year), and plots the transaction rate against brand quintile and the log sales share. Transaction rates are higher for products with larger market shares. This result can be rationalized in various ways; our model uses directed search to explain this pattern. Firms naturally search with more intensity for higher-valued brands.

Similarly, Figure 3b shows how the transaction rate changes with age. Here, we plot a standard smoothed hazard function to ask at what age a brand experiences its first transaction. We find that transactions peak between the ages of 15–20, around when sales peak. We next turn to the role of the transactions across firms, particularly looking at the interaction between leading and fringe firms.

3.3 Firm × Brand Analysis

The interaction between firms and brands, in particular through the firm-brands fit, can be informative for studying the implications of the transactions of brands across firms. This section focuses on the differences in sales and prices between fringe and leading firms both in general and in an event study exercise.

What is the effect of being held by a larger firm on log sales and log prices?[8] We use the regression in Equation (3) for our analysis:

$$y_{ikt} = \alpha_0 + \alpha_1 \mathbb{I}\{j(i) = \text{T10 firm}\} + \Gamma_{ik} + \Lambda_t + a_{t-b(i)} + \epsilon_{ikt}. \tag{3}$$

[8] Firm size is defined over the entire horizon of the data, but the results are similar if firm is defined by its size in the initial period. We define a large firm in our sample as a firm in the top 10 firms in a product group to include a broader set of transactions.

The regression evaluates an outcome variable y_{ikt} (e.g., log sales or log prices with product share weights of product i in group k at time t) with reference to whether the brand is held by a market leader. We include a product (brand-group) fixed effect in Γ_{ik}, a year fixed effect (Λ_t), and an age fixed effect $a_{t-b(i)}$.[9] Sales and prices are at the brand $\times$ product group level. The sales are simply the log of the sum total of sales revenue at the level of the brand. For prices, we take the sales-weighted log price for each unit-level product. This delivers a log price index, which is the geometric average of the price, for each branded product that can be observed before and after a brand transaction.

Table 4: Log Price and Sales Conditional on Holding Firm

	(1) Log Sales	(2) Log Price
Top 10 Firm Holding	0.391***	0.069**
	(0.000)	(0.019)
R^2	0.843	0.983
N	485,261	485,261

p-values in parentheses,clustered at brand $\times$ group level
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands. Data from 2007–2018. Source: USPTO and RMS Nielsen.

The results are striking on both counts, and hold true with various specifications. If a top 10 firm holds a brand, sales on average are 0.39 log points higher, whereas prices are on average 0.069 log points higher. Both sales and prices are higher at leading firms, indicating large firms could have both strategic and efficiency reasons to buy brands. We discuss these results further in the quantitative section when we ask about the overall effects of this brand reallocation on market activity.

Event Study: Impact of Reallocation on Sales and Prices. We observe brand transactions in the data and ask how prices and sales respond.[10] To ensure a relevant comparison group, we link transacted brands to never transacted brands with similar age, sales trends, and product group codes to the focal brands in this setting.[11] Both transacted brands and placebo brands are active for 7 years (3 years before event, event period, 3 years after), ensuring a balanced panel. We then run the regression,

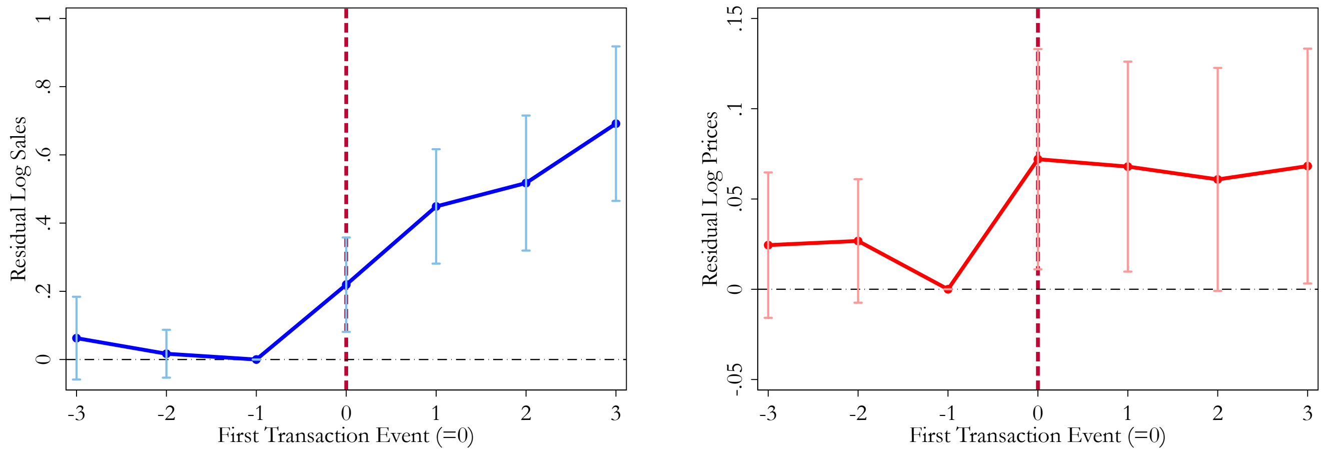
$$\log y_{it} = \alpha + \sum_{t=-3}^{3} \beta_t D_t \times reallocated + \lambda_t + \theta_i + \Lambda_a + \epsilon_{it}. \tag{4}$$

[9] In this specification, we use brand age as indicated by Nielsen but evaluate the robustness of these results to different specifications in Appendix B.3.

[10] We follow the same measurement of log sales and log prices in both the observed regressions and the event studies.

[11] We engage in a coarsened exact matching procedure following Blackwell et al. (2009) and is discussed in greater detail in Appendix B.2.

Figure 4: Coarsened Exact Match and Brand Transaction, Small to Large



(a) Sales × Transaction (b) Prices × Transaction

Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age, and year. 95% confidence interval standard errors clustered at the brand-group level.

Equation (4) illustrates the regression with the matched sample with D_t as the transaction indicator and β_t as our coefficient of interest, controlling for age (Λ), year (λ), and brand (θ). Figure 4 plots two separate regressions on one graph with different outcome variables of interest: log prices and log sales revenue.

After the event, both prices and sales move strongly, with sales moving more. With the increase in prices, the results in Figure 4 provide evidence that after adding additional brands, firms may increase their market power over time. Combining this finding with the rising rate of transfer from small to large firms can help connect the importance of brand dynamism with the aggregate distribution of markups across firms. Further, changes in markups is a key object of interest in our model. We now turn to summarize the three main facts discussed in this section.

Fact 1: Firm Level *Large firms disproportionately have more brands and more market share, and build their market share through brand reallocation more than brand creation.*

Fact 2: Brand Level *Brands build sales over time, and better and more mature brands are more likely to be reallocated across firms.*

Fact 3: Firm × Brand Level *When a brand is reallocated across firms, sales and prices both increase.*

Markets are concentrated, concentration is persistent over time, and concentration is built through reallocation and brand maturity (Fact 1). In line with findings on the product life cycle, we find patterns of higher profile and more mature brands being more likely to be reallocated (Fact 2). Directing more specific attention to brands across firms, we find increases in prices and sales upon transaction (Fact 3). These results motivate a model that can incorporate these forces to develop counterfactuals. Our model will incorporate these findings to characterize the drivers of concentration and the dynamic effects of brand reallocation. We turn to the model next.

4 Model

We introduce a firm dynamics model with brand appeal and reallocation of brand ownership. Leading firms hold multiple brands and compete against fringe firms, each with imperfectly substitutable branded products. In the model, each product is associated with a unique brand, corresponding to the unit of analysis in Section 3. Firms create brands, charge variable markups on their branded products, and buy and sell brand ownership in a secondary market. The model incorporates some standard features of an endogenous growth framework driven by product variety and product innovation. In addition to these standard features, we include three new ingredients: i) brand capital, ii) a brand or product life cycle, and iii) the secondary market for brands. Further, the model embeds the observed skewed distribution of firm size by focusing on the interaction between large and small firms. The model aims to provide a quantitative framework that incorporates the empirical facts of branding into a macroeconomic environment to evaluate the efficiency of brand reallocation and enable a rich set of policy counterfactuals.

Consumers choose varieties and supply labor to firms. Consumers have constant elasticity of substitution (CES) preferences across imperfectly substitutable products, and consumers prefer products with more brand appeal. The brand appeal of each product is driven by three sources of heterogeneity. Heterogeneity at the firm level determines the scope of the brand (e.g., through distribution or marketing). Heterogeneity at the brand level determines the consumer preference for the brand (e.g., through brand/product enjoyment). Heterogeneity at the firm $\times$ brand level determines the fit of the brand with the parent firm. The model incorporates these layers of heterogeneity in the reallocation market in a parsimonious way by leveraging tools from search theory (as in Menzio and Shi, 2011), simplifying the joint decision problem of firms. The key objects of interest in our model are the overall brand appeal and the distribution of ownership across firms. These will have important implications for both economic growth and market power.

Section 4.1 characterizes consumer demand for products, directing attention to the sources of brand appeal and how it interacts with consumer preferences. Section 4.2 focuses on the leading firms in each product group and the competition in each market. We then introduce the concept of trademarks and discuss the value of trademarks to a firm. Section 4.3 discusses the value of holding products for each firm and the innovation and reallocation decisions, with a particular focus on when reallocation is efficient versus strategic. Section 4.4 closes the model by completing the household problem. Section 4.5 characterizes the aggregation across product groups, the growth rate, concentration, and overall efficiency.

4.1 Product Demand

Time is continuous, and there is a representative household that endogenously supplies labor $\mathbf{L}_t$ and spends on products to maximize its discounted utility. We focus on the consumer's problem in this section and complete the full household problem in Section 4.4. At instant t, the real consumption of the household $\mathbf{C}_t$ is given by a Cobb-Douglas aggregator across a unit measure of product groups, indexed by $k \in [0, 1]$:

$$\ln \mathbf{C}_t = \int_0^1 \xi_k \ln C_{kt} dk, \tag{5}$$

where C_{kt} is the real consumption from product group k and ξ_k is the appeal of product group k to the household.

Each product group k at time t contains N_{kt} measure of imperfectly differentiable products each distinguished by a unique brand.[12] The real consumption from product group k, C_{kt}, is a CES aggregator across these products:

$$C_{k,t} = \left(Q_{kt}^{\frac{\nu-1}{\sigma_k}} \int_0^{N_{kt}} \psi_{ikt}^{\frac{1}{\sigma_k}} c_{ikt}^{\frac{\sigma_k - 1}{\sigma_k}} di \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \tag{6}$$

where c_{ikt} is the consumption on product i in group k, ψ_{ikt} is the brand appeal of product i in group k to the consumer, Q_{kt} is the total quality or appeal of products within the group (defined below in Equation 9), ν is the measure of love-of-variety, and $\sigma_k > 1$ is the substitution elasticity across products, which we allow to vary by product group.[13] We direct attention here to two key objects of interest in our model. First, we are concerned with the total number of products, N_{kt}, since more products contribute positively to consumer welfare. Second, we are concerned with each product's brand appeal, ψ_{ikt}. The joint distribution of these two objects connects directly to consumer welfare. Mirroring the empirical decomposition of product-level sales, we assume the appeal ψ_{ikt} is a combination of three components:

$$\log \psi_{ikt} = \alpha_{j(i,t)k} + \beta_{ikt} + \gamma_{ij(i,t)kt}. \tag{7}$$

Equation (7) closely resembles our empirical framework. $\alpha_{j(i,t)k}$ is the appeal of brand-owning firm j in group k, β_{ikt} is the specific brand appeal standing in for the consumers' taste for product i in group k, and

[12] We consider products as the goods sold to consumers and brands as the concept linking the product to an association in the consumer's mind, which can be held by firms. We abstract away from the mix of products under a general brand umbrella and focus on each product i in group k having a distinct associated brand in this section and paper.

[13] We introduce the love-of-variety ν so the model can target flexible levels of real consumption growth. This parameter does not directly affect the qualitative characterization of the equilibrium. For the model discussion, we set $\nu = 1$.

$\gamma_{ij(i,t)kt}$ is the match-specific quality between product i and its owning firm j in group k. The optimal consumption decision within group k gives the demand curve for variety i, given the appeal ψ and price p:

$$c_{kt}(p,\psi) = \psi \times p^{-\sigma_k} \times P_{kt}^{\sigma_k - 1}, \tag{8}$$

where the group-level price index P_{kt} is given by $P_{kt} = \left(\int_0^{N_{kt}} \psi_{ikt} p_{ikt}^{1-\sigma_k} di \right)^{\frac{1}{1-\sigma_k}}$. We normalize the aggregate price index to be 1 for any t. Given price p, the demand for product i increases in the appeal ψ. Given appeal ψ, product demand decreases in the price p to the degree that the price diverts from the group-level price index. The elasticity of substitution σ_k determines a consumer's sensitivity to price. In markets with high elasticity of substitution ($\sigma_k \to \infty$), individuals are very responsive to price and less responsive to appeal. In markets with low elasticity of substitution ($\sigma_k \to 1$), individuals respond more to the brand appeal.[14]

In a quantitative exercise, we allow all parameters to vary by group k. For the rest of the theoretical discussion, we perform two adjustments to aid exposition. First, we omit the group k index, as the group index does not change how we characterize the equilibrium. Second, we set $\xi_k = 1$ to study a general group k with group appeal 1.

4.2 Firms

Each product group contains one large multi-product firm and an endogenous measure of single-product firms. We refer to the multi-product firm as the group leader and single-product firms as the fringe firms. The leader and fringe firms differ in the following aspects: (1) *Capacity*. The leaders are able to own and operate many brands, whereas the fringe firms are only able to operate one brand. We denote the leader's basket of brands at time t as $\mathcal{I}_t^L$ and the fringe basket of brands as $\mathcal{I}_t^F$; (2) *Entry*. The leaders are not subject to firm entry and exit, while there is free entry of fringe firms. (3) *Productivity*. All products are produced using a linear technology in labor. The leader has productivity $e^{z/(\sigma-1)}$, and fringe firms have the same firm-level productivity 1. (4) *Pricing*. The leaders are big relative to their product group, and they internalize their impact on the group-level price index. The fringe firms are small relative to the market, and behave as monopolistically competitive firms. Each firm can charge a markup due to their brand appeal, but large firms have more pricing power through their larger appeal.

The sales of product i at instant t are determined by the following composite quality index (or appeal

[14] We study markets where goods are substitutable, e.g. $\sigma_k > 1$, which Hottman et al. (2016) find is a reasonable assumption in the consumer packaged goods market. In reality, some markets may produce a complementary bundle of goods (e.g., Davis et al., 2004 study this situation in the software industry, where leading monopolists may charge *lower* markups than fringe firms if their products are complementary). This is outside the scope of this current paper.

index) if it is operated by the leader j,

$$\log q_{it} = z + \alpha + \beta_{it} + \gamma_{ijt},$$

and the following composite quality index if it is operated by a fringe firm:

$$\log q_{it} = \beta_{it} + \gamma_{it}.$$

We define the quality index for the leader as the sum of quality indices across all of its brands, $Q_t^L = \int_{i \in \mathcal{I}_t^L} q_{it} di$, and the quality index of fringe firms as the sum of quality indices across all of their brands, $Q_t^F = \int_{i \in \mathcal{I}_t^F} q_{it} di$. Two group-level indices are welfare relevant in our model. The first index is the group-level composite quality index Q_t, because it increases the marginal utility of consumption through the love-of-variety:

$$Q_t = Q_t^L + Q_t^F.$$

We denote the growth rate of this object $g_t = \frac{\dot{Q}_t}{Q_t}$. The second is the ratio of the leader's quality index and the fringe firms' quality index:

$$\phi_t = \frac{Q_t^L}{Q_t^F}. \tag{9}$$

Competition. We assume the firms compete through prices and production and leading firms price their products jointly. For each product, leading firms internalize their impact on the group-level price index according to their market shares as in Atkeson and Burstein (2008), while fringe firms individually have no impact on the price index.

Trademarks. Whenever a firm introduces a new product, it has an incentive to trademark the product to protect the brand image. Further, there is evidence that firms trademark early in the firm life cycle (as noted by Dinlersoz et al., 2018), and trademark application costs are low. We thus study the resulting branded product and assume firms trademark upon innovation, making brand creation and product innovation a joint process. The corresponding market power from brand capital accrues to both leaders and fringe firms, but leaders can potentially markup higher than fringe firms. This pricing power is one force generating brand-ownership reallocation from small to large firms. The other force is the natural efficiencies of brands matching with the right firms.

Brand Creation. New brands are created through product innovation or brand creation, and both market leaders and fringe firms can innovate. The leader chooses its innovation intensity η by paying labor cost $D(\eta)$. $D(\eta)$ is increasing and convex in η, and $D(0) = 0$. The fringe firms can endogenously enter with an entry cost $\frac{\kappa^e}{Q_t}$. Brand creation from both fringe and leading firms leads to a new brand that the owning firm trademarks. New brands draw an initial brand-specific quality β from exogenous distribution $F_\beta(\beta)$ and match-specific quality γ from distribution $F_\gamma(\gamma)$.

Brand Life Cycle. When a brand is initially introduced, the brand has low appeal since consumers are not broadly aware of it. This is consistent with evidence on the long-run development of consumer demand presented earlier (and noted by, among others, Bronnenberg et al., 2009; Einav et al., 2021) and the previous evidence presented on low reallocation rates early in a brand's life cycle. Brands then develop customer capital through a dynamic process. This process induces both increases in sales and increases in visibility for reallocation of brand ownership. New brands draw initial appeal $\beta_0 \sim F_\beta$. We model the brand development process as, $d\beta = \iota(\bar{\beta} + \beta_0 - \beta)$. This equation relates the change in brand appeal $d\beta$ to the speed of maturity, ι, the potential brand appeal peak, $\bar{\beta}$, and the initial and current brand appeal, (β_0, β). In Section 6, we match ι and $\bar{\beta}$ to the life cycle in the data.

Brand Reallocation. The ownership of existing brands can be reallocated across firms within the same product group. We model this reallocation process as a market with search and matching frictions (consistent with observed gains from trade in ownership exchange as in David, 2020). We assume search is directed (consistent with the fact that brands of competitors are not hard to find) as in Menzio and Shi (2011). At each instant, buyers can create vacancies with a constant cost; Sellers post the transfers they would require in exchange for their brands; buyers, observing all posted transfers, direct search to their preferred sellers. The matching between sellers and buyers is modeled by a matching function that moves with the number of vacancies and the number of sellers. Mathematically, we assume the number of matches is given by $m(v, u)$, where v is the number of vacancies and u is the number of sellers. We assume m is increasing and concave in both arguments. It is useful to define the selling rate as $\lambda(\theta) = m(\theta, 1)$, where θ is the ratio between the number of buyers and the number of sellers.

Implications of Brand Reallocation. In the next section, we discuss brand reallocation in greater detail. Here, we briefly preview two extreme cases of brand reallocation: *efficient* and *strategic* reallocation. The intuition of these two polar cases captures important features of the market for brands. Firms reallocate the brand appeal associated with the brand β, which is the fixed component. However, reallocation may lead to different appeal through α (firm effect) or γ (firm $\times$ brand effect). In the case where the leading firm has large advantages (e.g., in distribution or marketing, $\alpha >> 0$), we expect transactions to exhibit efficiency gains since brands are allocated to a better firm. If leading firms do not have advantages in distribution and marketing (e.g., $\alpha \approx 0$), then the brand ownership transfer only exacerbates an appeal gap between leaders and fringe firms. We study this further when we characterize the market equilibrium.

4.3 Firm's Problem

Firms face a static pricing and production problem and a dynamic innovation and reallocation problem. We discuss these two problems in turn.

Static Pricing Problem. Fringe firms are infinitesimal relative to the market and thus do not internalize their impact on group-level price indices. In the equilibrium, they charge a constant markup $\frac{\sigma}{\sigma-1}$ in line with standard monopolistic competition models. The leaders have a different pricing problem because they are large relative to the market and internalize their impact on the group-level price index. Given the demand curve for each variety in Equation (8), the product-group leader's pricing decision is:

$$\max_{p_i} \int_{i \in \mathcal{I}_t^L} (p_i - e^{-z/(\sigma-1)} \mathbf{w}_t) c_t(p_i, \psi_i) di$$

s.t.

$$c_t(p, \psi) \text{ given by Equation (8).}$$

As in models in the patent race literature, the competition of a group can be summarized by the gap between leaders' quality and fringe firms' quality as noted in Equation (9). Given the gap ϕ, the equilibrium markup charged by the leader is increasing in its market share s, given by

$$\mu = \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1}, \tag{10}$$

and the market share depends on both the gap between leader and fringe firms and the markup,

$$s = \frac{\mu^{1-\sigma}}{\mu^{1-\sigma} + \phi^{-1} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}. \tag{11}$$

Equation (10) and Equation (11) constitute two equations in two variables (s, μ).[15] Given any gap ϕ, we denote the solution to this system of equations as $s(\phi)$ and $\mu(\phi)$. Due to the assumption of Cobb-Douglas aggregation across product groups, the leader's profit can be written as $\Pi(\phi(t))\mathbf{C}(t)$, where $\Pi(\phi)$ is the share of aggregate expenditure accruing to the leader,

$$\Pi(\phi) = \frac{s(\phi)}{\sigma\left[1 - s(\phi)\right] + s(\phi)}. \tag{12}$$

Similarly, the profit share that accrues in aggregate to fringe firms is

$$\pi(\phi) = \frac{1 - s(\phi)}{\sigma}. \tag{13}$$

[15] This derivation is discussed in Appendix C.3.

In Equation (12), Given a unit of expenditure, $s(\phi)$ accrues to the leader, while $1-s(\phi)$ accrues to fringe firms, who receive $\frac{1}{\sigma}$ profit margin. The leader, due to its collection of brand appeal, "perceives" a more inelastic consumer, $\sigma(1-s(\phi))+s(\phi)$, than fringe firms, σ. The inverse of the leader's perceived elasticity is its profit margin.

A leading firm's incentive to engage in brand creation and reallocation derives from its ability to increase its profits. We characterize the marginal increase of a leader's profit when it increases its quality gap from the fringe firms. We find this marginal profit has a closed-form solution in terms of market share, which we discuss in the following lemma.

Lemma 1 *The elasticity of profit with respect to a change in quality gap ϕ is*

$$\frac{\partial \log \Pi(\phi)}{\partial \log \phi} = 1 - s(\phi). \tag{14}$$

Two extreme cases are helpful in understanding the result in Equation (14). When the leader has 0 market share, the profit elasticity is 1. At this point, the leader has an infinitesimal share of the market and charges the same markup as the fringe firms. Thus a 1% increase in its market share translates into a 1% increase in profits without losing markup. When the leader has 100% of the market, the profit elasticity is 0. When the leader has taken over the whole market, a marginal increase in its quality gap only cannibalizes its own market share without changing firm-level profit.

Dynamic Innovation and Reallocation Problem. To characterize the incentives for product innovation and reallocation, we first write out the value of heterogeneous brands to leaders and fringe firms. For notational simplicity, we denote the vector of brand characteristics as $\mathbf{x}_{it} = \left(\alpha_{ij(i,t)k}, \beta_0, \beta_{it}, \gamma_{ij(i,t)}\right)$. Consider a brand currently operated by a fringe firm, with characteristics $\mathbf{x}$. To the fringe firm, this brand has value $u_t(\mathbf{x})$ that solves the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned}(\rho+g_t)u_t(\mathbf{x}) = & \underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\pi(\phi_t)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial u}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'}\Omega_t(\mathbf{x}',\mathbf{x}) - \theta\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{u}_t(\mathbf{x}).\end{aligned} \tag{15}$$

The value to a fringe firm in group k with brand appeal β and firm-brand fit γ has two components. The first component is the instantaneous return, which moves positively with β and γ. The second component is the option value in the search market. In this market, the fringe firm chooses search intensity θ to generate an arrival rate $\lambda(\theta)$ at which the firm receives the surplus from transferring its brand if the gains from trade are positive. If leaders have a considerable firm appeal advantage, α, they will demand a higher quantity of brands, shifting $\lambda(\theta)$ and inducing a higher value for the fringe firm through this channel.

The product group leader chooses its innovation and reallocation activity to maximize its discounted profit. At time t, given the innovation intensity η_t^L and the reallocation decision $\theta_t^{LF}(\mathbf{x})$ and $\theta_t^{FL}(\mathbf{x})$, the density of brands with characteristics $\mathbf{x}$ that are operated by the leader evolves according to:

$$\begin{aligned}\dot{n}_t^L(\mathbf{x}) = & \underbrace{\eta_t^L f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial n_t^L}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & \underbrace{- \lambda\left(\theta_t^{LF}(\mathbf{x})\right) n_t^L(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\mathbf{x}')>0} f_\gamma(\gamma)\lambda\left(\theta_t^{FL}(\mathbf{x}')\right) n_t^F(\mathbf{x}')d\gamma'}_{\text{F-t-L Reallocation}}.\end{aligned} \tag{16}$$

Similarly, the density of brands with characteristics $\mathbf{x}$ that are operated by fringe firms evolves according to:

$$\begin{aligned}\dot{n}_t^F(\mathbf{x}) = & \underbrace{\eta_t^F f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial n_t^F}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & \underbrace{- \lambda\left(\theta_t^{FL}(\mathbf{x})\right) n_t^F(\mathbf{x})}_{\text{F-t-L Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x}',\mathbf{x})<0} f_\gamma(\gamma)\lambda\left(\theta_t^{LF}(\mathbf{x}')\right) n_t^L(\mathbf{x}')d\gamma'}_{\text{L-t-F Reallocation}}.\end{aligned} \tag{17}$$

A product group leader, taking as given the entry decision of fringe firms, chooses its innovation and reallocation activity to maximize the discounted net profit,

$$\max_{\eta_t,\theta_t^{LF}(\mathbf{x}),\tau_t^{LF}(\mathbf{x})} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt,$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x})d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x})d\mathbf{x}},$$

$$B_t = \int \left[\lambda(\theta_t^{FL}(\mathbf{x}))\tau^{FL}(\mathbf{x}) - \theta_t^{FL}(\mathbf{x})\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}\right] n_t^F(\mathbf{x})d\mathbf{x},$$

$$S_t = \int \lambda\left(\theta_t^{LF}(\mathbf{x})\right)\tau^{LF}(\mathbf{x}) n_t^L(\mathbf{x})d\mathbf{x}.$$

The leader's problem is a complicated problem that involves the joint distribution of brand appeal and firm ownership. We show the full characterization of this problem in Appendix C.2. We find that this complicated problem can be characterized by calculating the discounted values of brands to different firms (leader or fringe). This result comes from two features of our model: (1). the brand appeal can be linearly added into a firm-level quality index $\int_{\mathcal{I}_t^L} q_{ijt}$; (2) the love-of-variety and competition of each product group are fully characterized by the quality indices.

For a brand with state $\mathbf{x}$ that is currently operated by the group leader, its discounted value to the leader is

$$(\rho+g_t)v_t(\mathbf{x}) = \underbrace{e^{z+\alpha+\beta+\gamma}\left(1+\phi_t\right)\Pi'(\phi_t)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial v}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ + \underbrace{\max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'}\left[-\Omega_t(\mathbf{x}',\mathbf{x})\right]^{+} - \theta\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}), \tag{18}$$

where $\Pi'(\phi_t)(1+\phi_t)$ is the flow marginal value and $U_k^L(\beta,\gamma)$ is the optimal value of selling to fringe firms. The (negative) value of a similar brand operated by the fringe to the leader is:

$$(\rho+g_t)y_t(\mathbf{x}) = -\underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\phi_t\Pi'(\phi_t)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial y}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} + \dot{y}_t(\mathbf{x}). \tag{19}$$

The value functions of the leader and fringe link to the reallocation decisions through the joint surplus, which we define as $\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right) = v_t(\mathbf{x}_L) - y_t(\mathbf{x}_F) - u_t(\mathbf{x}_F)$. $\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right)$ measures the joint surplus from trade for a brand reallocated from a fringe firm to a leader, with brand appeal β, fringe match quality γ', and leader match quality γ. Correspondingly, $-\Omega_t\left(\mathbf{x}_F,\mathbf{x}_L\right)$ is the joint surplus of reallocating the brand from a leader to a fringe firm. The joint surplus $\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right)$ satisfies the Bellman equation:

$$\begin{aligned}(\rho+g_t)\,\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right) = {} & \underbrace{\omega(\beta,\gamma_L,\gamma_F)}_{\text{Flow Gains from Trade}} + \dot{\Omega}_t\left(\mathbf{x}_L,\mathbf{x}_F\right) \\ & + \underbrace{\iota(\bar{\beta}-\beta_L)\frac{\partial \Omega}{\partial \beta_L}(\mathbf{x}_L,\mathbf{x}_F)}_{\text{Leader Maturity}} + \underbrace{\iota(\bar{\beta}-\beta_F)\frac{\partial \Omega}{\partial \beta_F}(\mathbf{x}_L,\mathbf{x}_F)}_{\text{Fringe Maturity}} \\ & - \underbrace{\max_{\theta^{FL}(\mathbf{x})} \lambda\left(\theta^{FL}(\mathbf{x})\right)\mathbb{E}_{\gamma'}\Omega_t(\mathbf{x}',\mathbf{x}_F)^{+} - \theta^{FL}(\mathbf{x})\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Fringe's Value of Selling}} \\ & + \underbrace{\max_{\theta^{LF}(\mathbf{x})} \lambda\left(\theta^{LF}(\mathbf{x})\right)\mathbb{E}_{\gamma'}\left[-\Omega_t(\mathbf{x}_L,\mathbf{x}')\right]^{+} - \theta^{LF}(\mathbf{x})\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Leader's Value of Selling}},\end{aligned} \tag{20}$$

where

$$\omega(\beta,\gamma_L,\gamma_F) = \left(\frac{e^{\alpha+\gamma_L-\gamma_F}+\phi}{1+\phi}\frac{\Pi}{\frac{\phi}{\sigma_k(1+\phi)}} - 1\right)\pi e^{\beta+\gamma_F}.$$

This equation provides the basis for gains from trade. We take note of two important polar cases in this environment. Table 5 focuses on the conditions under which reallocation would be *only efficient* or *only strategic*. We take the extreme cases in the parameter set that would induce completely efficient or completely strategic transactions.

Table 5: The Efficient and Strategic Gains from Reallocation

Case	Condition	Gains from Trade	Discussion
Efficient:	$\phi \to 0$	$(e^{\alpha+\gamma_L-\gamma_F}-1)\,\pi$	Gains from only $\alpha > 0$ or $\gamma_L - \gamma_F > 0$, sales $\uparrow$
Strategic:	$\alpha + \gamma_L - \gamma_F = 0$	$\left(\frac{\Pi}{\frac{\phi}{\sigma(1+\phi)}} - 1\right)\pi e^{\beta+\gamma_F}$	Gains from higher concentration, markup $\uparrow$

Efficient reallocation occurs when leaders have no market concentration. In this case, gains from trade emerge only from the leader expanding brand appeal or having a good fit with the brand (α or γ). The strategic reallocation occurs when the leader has no advantage in marketing or distributing the brand (they may even have a disadvantage, e.g., $\alpha + \gamma_L - \gamma_F < 0$), and thus the reallocation simply increases concentration. These two benchmarks provide useful conceptual extremes to characterize efficiency in the market. We now turn to the firms' decisions regarding innovation and reallocation.

Innovation Decisions. If there is positive brand entry for fringe firms, the expected value of brand creation must equal the entry cost κ^e adjusted by the wage-consumption ratio:

$$\mathbb{E}_\beta\left[u(\beta, 0)\right] = \kappa_e^F \frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{21}$$

Optimal innovation by the leader requires that the marginal cost of innovation equals the marginal benefit,

$$\mathbb{E}_\beta[v(\beta, 0)] = D'(\eta)\frac{\mathbf{w}_t}{\mathbf{C}_t}.$$

Reallocation Decisions. A central focus of our model is the brand reallocation flows across different firms. From Equation (15), the equilibrium buyer-seller ratio for a brand with quality (β, γ), where buyers and sellers are both fringe firms, equalizes the marginal value of trade and the marginal cost,

$$\lambda'(\theta_t^{FL}(\beta, \gamma))\mathbb{E}_{\gamma'}\Omega(\mathbf{x}', \mathbf{x})^+ = \kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{22}$$

The result is similar for leader-to-fringe reallocation,

$$\lambda'(\theta_t^{LF}(\beta, \gamma))\mathbb{E}_{\gamma'}\left[-\Omega(\mathbf{x}, \mathbf{x}')\right]^+ = \kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{23}$$

We expand on these details in Appendix C. Having characterized the innovation and reallocation decisions, we turn to closing the model with the household problem.

4.4 Closing the Model: Household Problem

To close the model, we detail the household's consumption-saving and labor supply decisions. The household can freely borrow or save by investing in a representative portfolio of firms in the economy, taking as given the interest rate and prices. This assumption means that all firm profits are returned to the household. We normalize the aggregate price index to be 1 and express other prices in their real units. Denote $\mathbf{w}_t$ as the real wage and $\mathbf{r}_t$ as the real interest rate. The household takes these prices as given and chooses its real consumption $\mathbf{C}_t$ and labor supply $\mathbf{L}_t$ to solve

$$\max_{c_{ikt},\mathbf{L}_t} \int_0^\infty e^{-\rho t}\left[\ln \mathbf{C}_t - \varphi_0 \frac{\mathbf{L}_t^{1+1/\varphi}}{1+1/\varphi}\right] dt,$$

s.t.

$$\dot{a} = \mathbf{r}_t a_t - \mathbf{C}_t + \mathbf{w}_t \mathbf{L}_t,$$

$$\mathbf{C}_t \text{ given by (5) and (6).}$$

The optimal saving decision implies the Euler equation must hold:

$$\frac{\dot{\mathbf{C}}}{\mathbf{C}} = \mathbf{r} - \rho,$$

and the optimal labor supply decision requires that the marginal rate of substitution between leisure and consumption equals the real wage:

$$\varphi_0 \mathbf{L}_t^{1/\varphi} = \frac{\mathbf{w}}{\mathbf{C}}.$$

Discussion of Cobb-Douglas Assumption. One key assumption from the household side simplifies the equilibrium. By assuming that the consumption from different product groups is aggregated through a Cobb-Douglas utility function, we assume that evolution within each product group does not lead to reallocation of market shares across product groups. In addition to being the standard assumptions in the patent race literature such as Liu et al. (2022) and in the product dynamics literature such as Hottman et al. (2016) and Argente et al. (2021), this assumption is without apology since our context is the product market at an annual frequency, where this reallocation across groups is small.

We discuss the evolution of the distribution in Appendix C. The main finding from the distribution is that the holdings of both the fringe and leading firms can be expressed in closed form, linking the flows across firms and innovation to aggregate market shares.

4.5 General Equilibrium and Aggregation

The model aims to provide a conceptual and quantitative framework that links branding activity to macroeconomic outcomes to quantify the implications for efficiency and welfare. This section discusses how innovation and reallocation in product markets lead to overall growth, concentration, and market efficiency.

Within-Group Equilibrium. We now reintroduce the notation for each product group k to discuss the equilibrium and aggregation. From the previous sections, we know the firm's optimal innovation and reallocation decisions and maturity process shape the aggregate appeal,

$$Q_{kt} = \int_{i \in \mathcal{I}_{kt}^L} q_{ikt} di + \int_{i \in \mathcal{I}_{kt}^F} q_{ikt} di.$$

Given the detrended distribution of appeal and the innovation and reallocation rates, the growth rate of total quality or appeal within product group k is

$$g_{kt} = \underbrace{\eta_{kt}^L + \eta_{kt}^F}_{\text{Innovation}} + \underbrace{\iota_{kt}^L + \iota_{kt}^F}_{\text{Maturity}} + \underbrace{\Lambda_{kt}^{FL} + \Lambda_{kt}^{LF}}_{\text{Reallocation}}, \tag{24}$$

where

$$\Lambda_{kt}^{FL} = \int_{\Omega_t(\mathbf{x}',\mathbf{x})>0} \exp(\alpha_k + z_k + \gamma' - \gamma) \lambda_t^{FL}(\mathbf{x}) n_t^F(\mathbf{x}) d\mathbf{x},$$

$$\Lambda_{kt}^{LF} = \int_{\Omega_t(\mathbf{x},\mathbf{x}')<0} \exp(\gamma' - \gamma - \alpha_k - z_k) \lambda_t^{LF}(\mathbf{x}) n_{kt}^L(\mathbf{x}) d(\mathbf{x}),$$

$$\iota_{kt}^L = \iota \int \exp(\bar{\beta} - \beta) n_{kt}^L(\mathbf{x}) d\mathbf{x},$$

$$\iota_{kt}^F = \iota \int \exp(\bar{\beta} - \beta) n_{kt}^F(\mathbf{x}) d\mathbf{x}.$$

η_{lt} and η_{ft} are the endogenous leader and fringe innovation decisions. ι_{kt} is the maturity process at the group level. Λ are the respective flows in each direction (FtL, LtF), which is a function of the distributions of α, β, and γ. We are now ready to define the group-level equilibrium.

Definition 2 *(Group Equilibrium) A group equilibrium in group k given the aggregate wage-GDP ratio $\frac{\mathbf{w}_t}{\mathbf{C}_t}$ is $\{\phi_{kt}, g_{kt}\}$ and $\{v_{kt}(\mathbf{x}), u_{kt}(\mathbf{x}), \Omega_{kt}(\mathbf{x}_L, \mathbf{x}_F), \theta_{kt}^{LF}(\mathbf{x}), \eta_{kt}^L, \eta_{kt}^F\}$, and $\{n_t^L(\mathbf{x}), n_t^F(\mathbf{x})\}$ such that*
1. Given (ϕ_{kt}, g_{kt}), $\{v_{kt}(\mathbf{x}), u_{kt}(\mathbf{x}), \Omega_{kt}(\mathbf{x}_L, \mathbf{x}_F), \theta_{kt}^{LF}(\mathbf{x}), \eta_{kt}^L, \eta_{kt}^F\}$ solve firms' optimization;
2. Given Step 1, $\{n_t^F(\mathbf{x}), n_t^L(\mathbf{x})\}$ solves equations (16) and (17);
3. (ϕ_{kt}, g_{kt}) are consistent with equations (18) and (24)

The group-level equilibrium can be solved in isolation from the aggregate variables working from step 1 to step 3. With the group-level equilibrium characterized, we turn to the overall brand appeal in the economy and consumer welfare.

This section links the outcomes in each market k to the overall efficiency in the economy. The results in this section inform the eventual discussion of aggregate efficiency and the social planner's problem. Given the partial equilibrium within each market, specifically $\{\phi_k, Q_k\}$, the following proposition summarizes the general equilibrium of the economy.

Proposition 3 *Given the product-group equilibria, the general equilibrium of the economy is characterized as follows:*

1. Given $\{\phi_k, Q_k\}$, calculate the following productivity, markup, and misallocation indices:

(Productivity)

$$\mathbf{Q}(t) = \mathbf{Q}(0)\exp(\mathbf{g}t), \qquad \mathbf{g} = \int_0^1 \frac{\xi_k \nu}{\sigma_k - 1} g_k dk; \tag{25}$$

(Markup)

$$\mathbf{M} = \exp\left(\int_0^1 \xi_k \log M_k dk\right), \tag{26}$$

$$M_k = \left[\frac{\phi_k}{1+\phi_k}\mu_k(\phi_k)^{1-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}}; \tag{27}$$

(Misallocation)

$$\mathbf{A} = \int_0^1 \xi_k \left(\frac{M_k}{\mathbf{M}}\right)^{-1} A_k dk, \tag{28}$$

$$A_k = \frac{\phi_k}{1+\phi_k}\left(\frac{\mu_k(\phi_k)}{M_k}\right)^{-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{-\sigma_k}. \tag{29}$$

2. The aggregate objects $\mathbf{C}, \mathbf{L}_P, \mathbf{L}_S, \mathbf{L}_D$ are given by

$$\mathbf{C} = \mathbf{QAL}_P, \tag{30}$$

$$\mathbf{w} = \frac{\mathbf{Q}}{\mathbf{AM}}, \tag{31}$$

$$\varphi_0(\mathbf{L}_S + \mathbf{L}_P + \mathbf{L}_D)^{1/\varphi} = \frac{1}{\mathbf{AML_P}}. \tag{32}$$

This aggregation result follows a similar structure to recent literature on the aggregate implications of firm-level markups (or markdowns) such as Edmond et al. (2015) and Berger et al. (2019).

Definition 4 *(General Equilibrium) A general equilibrium is $\frac{\mathbf{w}_t}{\mathbf{C}_t}$ such that:*

1. All product groups are in equilibrium as defined in Definition 2;

2. Given the group equilibrium, the aggregation holds as defined in Proposition 3.

Our welfare metric is the discounted utility of the representative household:

$$\mathcal{W} = \int_0^\infty e^{-\rho t} \left(\ln \mathbf{C}_t - \varphi_0 \frac{\mathbf{L}^{1+1/\varphi}}{1+1/\varphi} \right) dt. \tag{33}$$

This aggregation relates the two main aspects of our quantitative analysis. First, we evaluate the aggregate appeal of the economy, **Q**, which moves positively with social welfare. Second, we evaluate the misallocation, **A**, and firm markups, **M**, which move negatively with social welfare. This general framework operates in the background of our estimation and quantitative analysis.

4.6 Discussion of the Model

Before turning to the quantitative analysis, we stress some specific contributions of the model. One salient feature of product market competition is multi-product firms and heterogeneous products (e.g., as noted in Hottman et al., 2016). In our quantitative analysis, these features matter for our conclusions. Here, we highlight the connections and departures of our theoretical model with prior literature in two domains.

Relationship to Endogenous Growth Theory. Dealing with a firm's holding of heterogeneous products or brands in a variable markup environment is a daunting task because it involves a multi-dimensional portfolio choice decision. This is different from the step-by-step innovation models widely used in the literature (Aghion et al., 2001; Klette and Kortum, 2004; Akcigit and Kerr, 2018; Peters, 2020; Cavenaile et al., 2021), where the relevant firm-level technology measure is the summation of past innovation steps. By making this assumption, this class of models assumes that once an innovation is incorporated into the firm, the specific innovation quality is no longer relevant. This feature is not well suited to our context, because brands are born heterogeneous and experience long life cycles (see Figure 2); further they have different chances of being reallocated depending on their age and sales (see Figure 3).

Once we consider this heterogeneity and its impact on firms' dynamic decisions in a discrete step-by-step innovation model, the firm's problem involves tracking an endogenous distribution of brand characteristics. This is an infinite-dimensional object and is complicated to tackle (e.g., noted in finance as in Merton, 1973 and macroeconomics as in Krusell and Smith, 1998). Our assumption of infinitesimal products greatly simplifies this problem while maintaining the rich heterogeneity. In our setting, the decision regarding each brand can be analyzed separately, with the firm's evaluation of its full basket of brands summarized by the quality gap ϕ.

Killer Acquisitions. In the model, large firms have an incentive to buy brands even without efficiency gains, since they can clear out competitors and increase markups. This section considers the *protective* incentive of a leader to maintain market concentration. Our model thus speaks to the literature on how this protective incentive leads to killer acquisitions (Cunningham et al., 2021). A killer acquisition is a case where large firms buy up-and-coming brands (or firms) to kill the brand. Our model provides an interesting threshold that links to this general phenomenon.

We define a "killer acquisition threshold", $\bar{s}$. This threshold $\bar{s}$, defined in Equation (34), is the market share beyond which the protective incentive of the leader implies they have an incentive to engage in acquisitions *even if the fringe firm is arbitrarily more efficient at holding the brand.*[16] For any $s > \bar{s}$, there will be acquisitions of any fringe brand regardless of the fringe firm's efficiency. Further, a leader will *never* sell their brand to a fringe firm regardless of how much more efficient the fringe firm is if their market share is greater than in Equation (34),

$$\bar{s} = \frac{\sigma}{2\sigma - 1}. \tag{34}$$

Equation (34) provides some interesting benchmarks. When products are highly substitutable ($\sigma \to \infty$), the threshold $\bar{s}$ converges to 1/2. This means that if the leading firm has over 50% of the market, they will *never* sell a brand, regardless of the efficiency differential. Killer Acquisitions in our model is a case where the leader purchases a product to decrease its sales. Taken to a limit $\sigma \to \infty$ is a killer acquisition as a firm buys the product without any incentives to produce. This result is due to firms internalizing the lost market share of efficient competitors. This substitution elasticity is likely close to the situation noted in Cunningham et al. (2021), who study the market for pharmaceuticals, where the question about up-and-coming drugs is more about the underlying biochemistry than brand image.

In the case where products are highly differentiable ($\sigma \to 1$), the role for killer acquisitions disappears ($\bar{s} \to 1$). This highlights an important point on product differentiation. When there is significant differentiation, firms have an interest in sorting brands to efficient firms, and leading firms would not want to kill the brand image of a product, as it destroys brand capital. These forces speak to a central tension in our environment.

5 Estimation

We estimate the model parameters employing the empirical moments of brand creation, maturity, and reallocation. The model delivers simple objects that enable identification and estimation. We primarily explore two methods of estimation.

[16] Appendix C.4 discusses the derivation of this threshold in detail.

5.1 Estimation Procedure

In the baseline estimation, we assume all product groups are identical in their parameters, and we estimate the model to match the aggregate moments. We refer to this as the *homogeneous group estimation*. The homogeneous group estimation provides a natural benchmark, and our primary results will be understood through this lens.[17]

We then estimate the model assuming product groups are heterogeneous in their substitution elasticity, search cost, and entry cost. We refer to this estimation as *heterogeneous group estimation*. This approach provides a more granular analysis of particular product groups. In the following paragraphs, we detail the homogeneous group estimation procedure.[18] Table 6 provides the link between moments in the data that are independently calibrated and jointly estimated to the relevant parameters in the homogenous group case.

Externally Calibrated Parameters. We set the discount rate to be the annual risk-free rate of $\rho = 0.02$. We set the labor supply elasticity to be $\varphi = 0.5$ (Berger et al., 2019), and the innovation elasticity to be 1 ($D(\eta) = \kappa_e^L \eta^2$), as in Akcigit and Kerr (2018). Hottman et al. (2016) estimate the substitution elasticities in a demand system similar to our setting. We thus directly take the estimates of the substitution elasticities from Hottman et al. (2016). In the homogeneous group estimation, we set the substitution elasticity $\sigma = 6.9$, which is the median substitution elasticity from Hottman et al. (2016). For the heterogeneous group case, we take Hottman et al. (2016) group-level substitution elasticities.

Estimation of Search and Innovation Costs. We estimate the innovation and search costs to match observed innovation rates and reallocation rates. Three cost shifters exist that we allow to vary by product group: the innovation cost shifter d_k, the entry cost κ_k^e, and the search cost κ_k^s. Our model provides a direct link from observed market shares and new brand creation rate at the group level to these costs.

Estimation of Leader Appeal Advantage. We reference the regression from Equation (3) in Section 3.3 to estimate the sales difference between a leader and fringe firm in holding a brand, which is 0.391. Controlling for brand $\times$ group, age, and year fixed effects, top 10 firms have 0.391 log points higher sales than non-top 10 firms.

Estimation of Gains from Trade. We evaluate event studies when brands flow from large to small firms to identify the distribution of the fit between brand and firm when there are gains from trade. We

[17]We discuss the details and estimation methods in Appendix D.

[18]The estimation process is very similar for heterogeneous groups, but takes different inputs for each ingredient. We include the heterogeneous group analysis in our results and discuss the procedure further in Appendix D.

assume the distribution of γ, F_γ, is exponential with the mean identified off of the event studies. We find this average to be 0.17.

Table 6: Estimation Moments and Parameters

Parameter		**Value**	**Moment**	**Data (p.p.)**	**Model (p.p.)**
Independently Calibrated					
Household Parameters					
Discount Rate	ρ	0.02	Annual Risk-free Rate	**Exact Match**	
Substitution Elasticity	σ	6.90	Hottman et al. (2016)		
Firm Parameters					
Leader Advantage	α	0.39	Leader Advantage (Table 4)		
Product Quality				**Exact Match**	
Age Profile – Growth Rate	ι	0.04	Sales growth (Figure 2a)		
Age Profile – Peak Appeal	$\bar{\beta}$	0.45	Sales peak (Figure 2a)		
Distribution at Entry – $\mathbf{N}(0, \varsigma_{\beta_0})$	ς_{β_0}	2.31	SD sales, age 0 (Figure 2b)		
Innov. + Reall. Elasticities				**Exact Match**	
Matching Elasticity	m	0.21	Sales-Reallocation Profile		
Innovation Elasticity	d	1.00	Akcigit and Kerr (2018)		
Jointly Estimated					
Leader Innovation Cost	κ_e^L	5232.20	Leader New Product Share	0.25	0.25
Fringe Innovation Cost	κ_e^F	7.01	Fringe New Product Share	0.83	0.83
F-t-L Reallocation Cost	k_s^{FL}	624.67	F-t-L Flows	0.62	0.62
L-t-F Reallocation Cost	κ_s^{LF}	14.29	L-t-F Flows	0.53	0.53
Match Quality	Exp. Dist $\bar{\gamma}$	0.13	L-t-F Sales Effect	0.17	0.17

Notes: Parameters estimated separately (top panel) and jointly (bottom panel). Source: RMS Nielsen, USPTO and author calculations.

We rely on the optimality conditions for innovation, entry, and acquisition to recover these parameters. First, we note the marginal value of the state variables can be written as functions of the quality gap ϕ and growth rate g. Both variables have data counterparts. Specifically, the quality gap ϕ has a one-to-one mapping to the observed market share given σ_k; the growth rate g is linked to the brand creation rate by the fringe firms. With these two variables, we can directly calculate the marginal value of brands for the group leader. For each product group, we find the set of parameters (κ_k^e, κ_k^s) that minimize the distance between the data and the model's prediction of the leader's innovation rate, average selling rate, and innovation rate of fringe firms.

Estimation of Matching Elasticity. We estimate the innovation and matching elasticity using indirect inference. The targeted moment for this elasticity is the age profile of a brand getting transacted. In our model, the difference between the transaction rate for a new brand and for a mature brand is governed by the difference in marginal benefits and the matching elasticity. The difference is in the matching elasticity. In the extreme, if the matching function is inelastic with respect to tightness, no differential in the sales-transaction rates exists. Our estimation yields a matching elasticity of 0.292.

5.2 Comparison of Untargeted Moments

We now compare our model's predictions to untargeted moments from the data to evaluate the overall model fit to relevant outcomes in the data. We discuss five different moments of interest, which are detailed in Table 7.

Out-of-Sample Summary. Table 7 summarizes the untargeted moments in our analysis and indicates the sign of the response if there is not a point estimate. Qualitatively, our out-of-sample moments match the data and the literature.

Table 7: Untargeted Moments, Summary

Outcome of Interest	Model	Data/Literature
Leader Market Share	0.327	0.316
Event Study Log Prices	0.066	0.057
M&A Premium	0.42	0.47 (David, 2020)
Entry Response to Transactions	Qualitative Match (+)	0.028
Entry Response to Transaction Age	Qualitative Match (-)	-0.083
Age Distribution of Brands	see Figure 5	see Figure 5

Market Concentration. We do not directly target market concentration, e.g. the sales share of the market leader. Our model focuses on the observed innovation, reallocation, and maturity in the data to predict a given market share for the leader. However, many forces determine a market leader's share. In the estimated model, we predict that the leader will hold 32.7% of the market. In the data, the leader holds on average 31.6% of the market in each group, which is close to the model prediction.

Fringe-to-Leader Event Study: Prices. The model delivers an out-of-sample prediction on the change in prices upon transaction from a fringe to a leading firm. We follow Hottman et al. (2016) who find that average marginal costs for leading firms are similar to fringe firms, and set leader labor productivity $\exp(z) = 1$. We then predict the change in brand prices upon transaction, and find a similar response in the quantitative framework (0.066) to our event study (0.057).

M&A Premium. How the gains from trade are split between buyers and sellers of brand ownership is important for counterfactual analysis. We compare our model's prediction regarding the rent splitting with the rent shares observed in the data. In the models with random search (David, 2020), the rent split between buyers and sellers is primarily determined by the bargaining powers of both parties in the Nash bargaining step. In our model, due to the assumption of competitive search, the rent split is a by-product of the matching process and thus primarily determined by the estimated matching elasticity m. Employing detailed M&A data, David (2020) estimates the average premium to be 0.47. Our model predicts a weighted premium of 0.42, which is in line with this empirical finding.

Entry, Reallocation, and Maturity. While we separately used moments in entry, reallocation, and maturity in generating the model, we did not use any features of the *correlation* between these movements. Here, we discuss some qualitative out-of-sample results. First, we expect firms to be more likely to enter if the firm expects a brand reallocation from a leading firm. Second, we expect this to be stronger in markets where brands are reallocated at younger ages. We perform the following logistic regression to understand the contribution of transaction age and transaction rate at the group-year level to the effect of brand entry, regressing an indicator on whether a brand is entering a product group on the mean transactions in the group by year $\bar{M}_{jt}$, and the average age of transactions $\bar{D}_{jt}$ by group-year;

$$\eta_{ijt} = \beta_0 + \beta_1 \bar{M}_{jt} + \beta_2 \bar{D}_{jt} + \epsilon_{ijt}. \tag{35}$$

Table 8: Logistic Regression of Probability of Holding Entering Brand and Transaction Rate/Age

	(1) Brand Entry	(2) Brand Entry	(3) Brand Entry
Transaction Rate (Standardized)	0.011		0.028***
	(0.0074)		(0.0075)
Transaction Age (Standardized)		-0.079***	-0.084***
		(0.0067)	(0.0068)
N	367,421	367,421	367,421
Pseudo-R2	0.0135	0.0142	0.0142

Standard errors in parentheses.

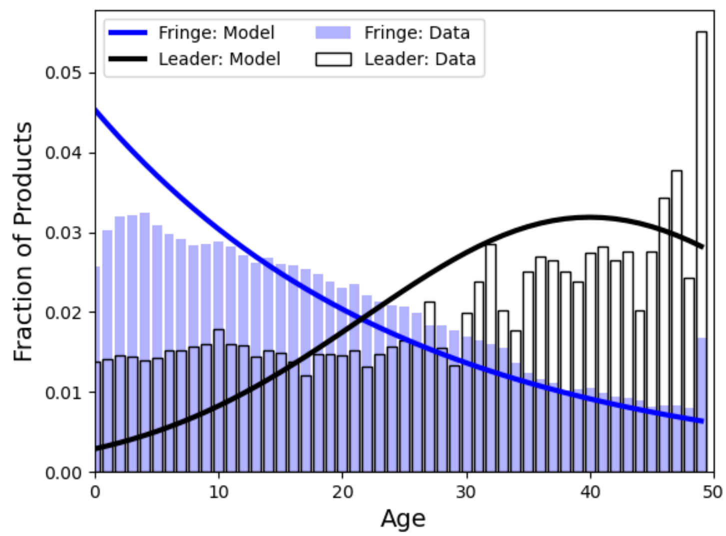
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Brand entry logistic regressions on reallocation rates and age following Equation (35). Source: RMS Nielsen and USPTO.

We find a weak correlation between entry and reallocation, shown in column (1) in Table 8—part of the reason for the weak connection links to maturity. In markets where older brands are transacted, we would expect the amount of transactions to have less of an impact on entry. When we control for both transaction rate and the average transaction age, we find a strong effect of each independent variable. We direct attention to column (3), which includes both the transaction rate at the group-year level and transaction age at the group-year level. We find that both forces independently matter for brand entry, consistent with the model predictions. This provides an important insight for policy analysis. Policies that direct attention to the costs of reallocation need to be aware of how this interacts with the entry margin.

Age Distribution of Brands. Due to selective reallocation and differences in innovation intensities, the age distribution of brands held by leaders and fringe firms are likely to differ. While untargeted, this moment provides affirmation that the reallocation and innovation margins are correctly generating the age distribution of brand holdings. We find that the model and data indicate a very similar pattern. In the data, we see the age distribution of holdings. Leaders tend to hold older brands, as indicated in Figure 5.

Figure 5: Age Distribution of Brand Holdings



Notes: Age Distribution of leading (top firm) and fringe firms (all else), in data and model. Source: Author calculations and USPTO.

6 Quantitative Analysis

With the estimated model, we are ready to explore the quantitative implications of our framework. We do this in two steps. First, we decompose the main forces driving the variation in growth and concentration and discuss welfare implications. Second, we explore various policy counterfactuals related to innovation, reallocation, and antitrust policies.

We start by discussing the sources of growth (innovation, maturity, reallocation) through the lens of our model. We then turn to the sources of market concentration. These two forces present an important tension in the economy, and our policy analysis will explore this tension. On the second point, we analyze standard policies (e.g., blocking acquisitions, acquisition taxes and subsidies, entry subsidies) through the lens of our quantitative framework and evaluate their joint effect on growth, concentration, and consumer welfare. We focus on the main characterization under the homogeneous group estimation.

6.1 Sources of Growth: Innovation, Maturity, and Reallocation

Most growth models from expanding product variety focus on product entry as the central driver of economic growth.[19] Motivated by empirical evidence from Section 3, we depart from this standard and note that, in addition to entry, reallocation and maturity are also essential components of growth. We return to our growth equation from the model to explore the interaction between these margins. From the model, the growth rate of real consumption can be decomposed into the following margins:

$$g_C = \int_0^1 \frac{\xi_k \nu}{\sigma_k - 1} \left(\underbrace{\eta_{kt}^L + \eta_{kt}^F}_{\text{Innovation}} + \underbrace{\iota_{kt}^L + \iota_{kt}^F}_{\text{Maturity}} + \underbrace{\Lambda_{kt}^{FL} + \Lambda_{kt}^{LF}}_{\text{Reallocation}} \right) dk,$$

where ξ_k is the appeal of group k, σ_k is the substitution elasticity of group k, and ν is the love-of-variety of the representative consumer. Our goal is to use the growth equation to decompose the variation driven by the three main processes in the data. First, there are the innovation rates of the leader and the fringe, η. Second, there is brand maturity, ι. The last two are the reallocation flows, Λ: fringe-to-leader and leader-to-fringe.[20]

Figure 6 focuses on the contributions to growth in three different scenarios applying the homogeneous group calibrated model.[21] First, we evaluate the baseline economy (red), then we turn off fringe-to-leader reallocation (blue).

We stress two main findings from Figure 6. First, as can be seen in the baseline economy, fringe entry η_F, brand maturity, and fringe-to-leader reallocation, Λ^{FL}, are the three most prominent sources of growth. Overall, steady-state real consumption growth is 2.1%, which targets the expenditure growth of the consumer product group as measured in the data over the time period. Out of this, 0.8% comes from fringe entry and 0.2% comes from leader entry. As a result, more than half of the growth in the quantitative framework is from the new components of brand dynamics: maturity and reallocation. Maturity contributes to growth of 0.8%, and reallocation on its own contributes to growth of 0.3%, though reallocation also has indirect effects through entry and maturity in equilibrium.

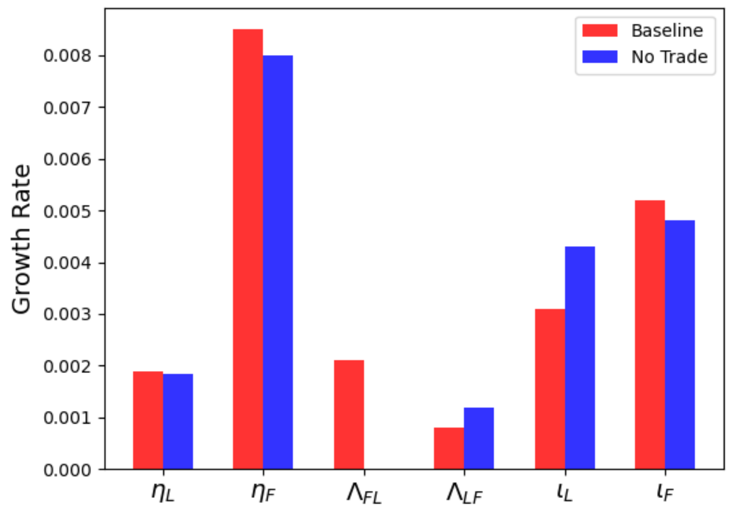
Second, when we shut down reallocation across firms, the steady-state response of each force is small. Overall fringe innovation rate declines by 10%, and the growth from reallocation from fringe-to-leader (Λ^{FL}) is shut down completely. However, this does not include the transitional dynamics since the economy is operating at a lower capacity. We discuss the implications for overall welfare in Section 6.4.

[19]In this section, we refer to this as product innovation or product creation interchangeably.

[20]We apply the definitions of leader and fringe discussed in Section 3 and the theoretical specifications from Sections 4 and D.

[21]Due to the log-linear structure of the utility function, the growth rates at the product-group level can be decomposed and linearly aggregated.

Figure 6: Sources of Growth, Baseline and No Reallocation Equilibrium



Notes: Sources of growth from fringe F, leader L; innovation (η), reallocation (Λ), and maturity (ι). Source: Author calculations.

6.2 Sources of Concentration: Innovation, Maturity, and Reallocation

As this paper has stressed, concentration emerges from three dynamic forces. Within the model, concentration can be decomposed into the dynamic components driving it,

$$\phi_k = \frac{\eta_L + \iota_L + \alpha\Lambda^{FL} - \Lambda^{LF}}{\eta_F + \iota_F - \Lambda^{FL} + \gamma\Lambda^{LF}}. \tag{36}$$

For each type of flow, we calculate the ratio between the predicted concentration and baseline concentration, where we use the leader share. Table 9 focuses on the contributions of innovation and reallocation to concentration.

Table 9 compares the leader's market share in the baseline economy to the counterfactual case where there is no reallocation and no maturity on the balanced growth path in the homogeneous group estimation. We thus report the leader's market share and the contributions of innovation, maturity, and reallocation to this market share by assuming the corresponding elements are zero. Ignoring the maturity margin, our model predicts a higher concentration (from 32.70% to 36.83%). Leaders tend to hold more mature brands that grow slower, while fringe firms tend to hold new brands that grow faster, matching the pattern in Figure 2a. Shutting down both reallocation and maturity decreases concentration to 27.37%. In this case, reallocation contributes to one-forth of the baseline leader's market share.

Simply using Equation (36) ignores the equilibrium responses of firms' strategies. In the second row of Table 9, we report the counterfactual leader's market share by solving the new balanced growth path without reallocation. Comparing to the baseline level, in the balanced growth path without reallocation,

Table 9: Concentration — Innovation vs. Reallocation

	All	Innovation + Reallocation	Innovation Only
a. Homogeneous Groups			
Baseline			
Leader's Market Share (%)	32.70	36.83	27.37
No Reallocation			
Leader's Market Share (%)	21.59	24.27	28.44
a. Heterogeneous Groups			
Baseline			
Leader's Market Share (%)	32.70	35.92	27.12
No Reallocation			
Leader's Market Share (%)	21.33	22.78	27.87

Notes: Each cell reports the leader's market share. For the column All, we report the leader's market share in the baseline case and the case without reallocation, allowing all components of Equation (36) to change; For the column Innovation+Reallocation, we assume maturity rate $\iota = 0$, allowing innovation and reallocation to change; In the column Innovation Only, we only allow the innovation rate to change. Source: Author calculations.

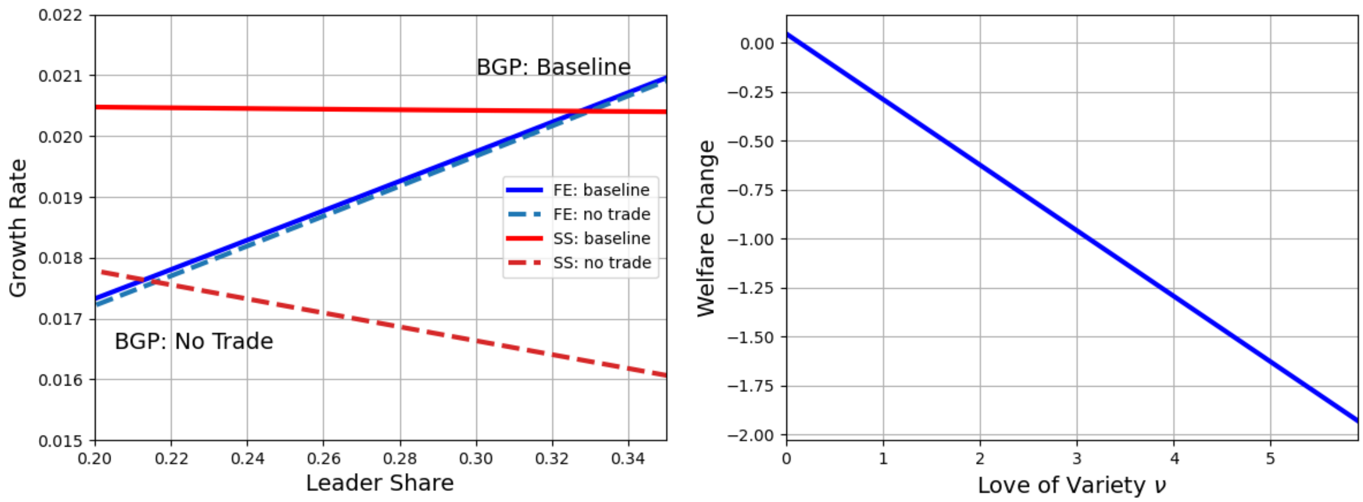
the leader's market share falls to 21.59%. However, if we only focus on the predicted concentration due to innovation, concentration increases without reallocation. This comes from the rent-sharing effect of reallocation. When reallocation is shut down, fringe firms are less compensated by the option value of selling, and fringe firms' entry rate falls. This fall increases concentration on the balanced growth path. As a result of both the direct effect (less reallocation) and indirect effect (less entry), shutting down reallocation decreases concentration by over 30%.

6.3 Good Concentration or Bad Concentration?

Recent literature (e.g., Covarrubias et al., 2019) points out that concentration can be "good" when more productive firms take larger market shares and increase consumer welfare, but it can also be "bad" when larger firms amass pricing power and restrict consumer substitution. Our quantitative results point to the coexistence of both effects. In our baseline estimation, the growth effect dominates and the reallocation of brand ownership tends to increase aggregate efficiency. We note here that this conclusion hinges on the magnitude of the love-of-variety. By increasing the love-of-variety elasticity ν, we up-weight the importance of consumption growth. An informative threshold is the level of love-of-variety elasticity such that the growth effect and the concentration effect exactly offset each other. Figure 7 illustrates this tradeoff, plotting the equilibrium growth rate against the concentration calibrated from the homogeneous group model. In this scenario, we plot two steady-state curves: one in the benchmark equilibrium and one without any fringe-to-leader brand exchange.

There are countervailing effects on growth from rising concentration. A more concentrated industry ($\phi \uparrow$) is associated with more opportunity from entry through reallocation and higher markups from incumbents (free entry curve). However, given a level of brand appeal in the economy, a higher concen-

Figure 7: Welfare Comparison to No-Reallocation BGP



(a) S.S. Comparison, Growth (g) and Leader Share

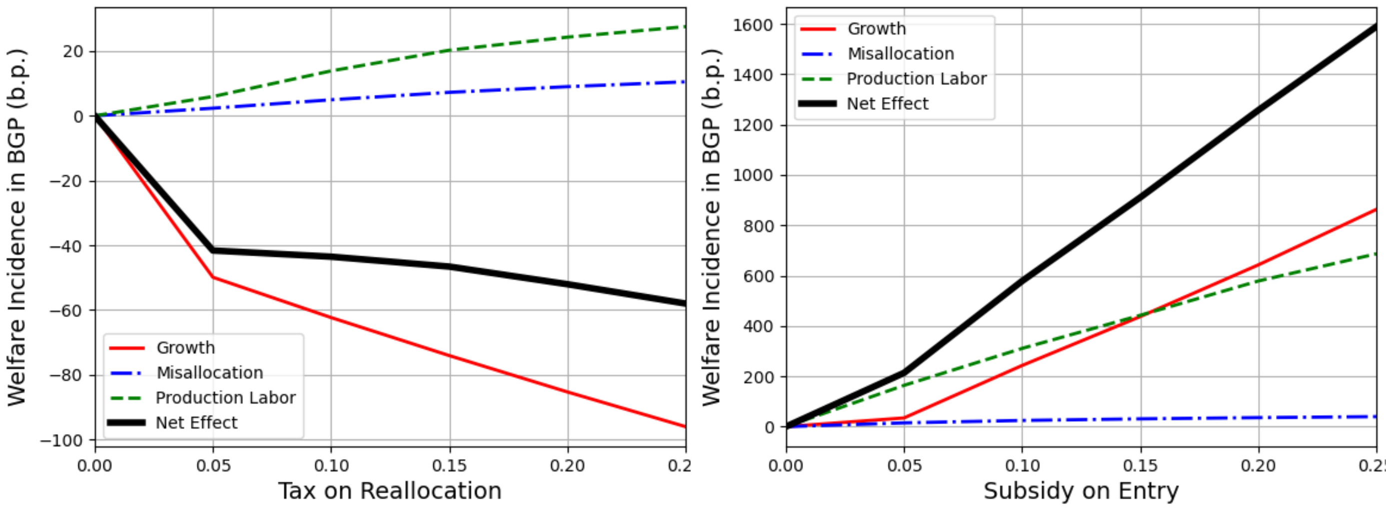
(b) Varying Love-of-Variety ν

Notes: Panel (a) plots the counterfactual scenarios for concentration and growth under no reallocation which is governed by the free entry curve and steady-state curve; Panel (b) plots the welfare loss from shutting down reallocation, through varying love-of-variety. Source: Author calculations.

tration will dampen growth by limiting the ability of entrants to build brand appeal and due to leaders innovating less (steady state curve). Further, higher concentration gives leaders more pricing power and inefficiency from markups. However, in net, the reallocation is efficient. Shutting down brand reallocation decreases steady state welfare by 1.93% given the estimate of love-of-variety (5.9) that links expenditure growth to consumption growth.

The love-of-variety question connects to classic debates about the role of brands and marketing in the economy (e.g., the role of advertising and branding as a costly or informative expenditure, Galbraith, 1958 versus Stigler, 1961), an issue we mostly sidestep in this paper. Since brand appeal and brand reallocation continues to play a large role in the macroeconomy, this issue has important implications for considering the nature of brand ownership acquisitions. If an increase in sales at the consumer level is due to the consumers love-of-variety, this leads to more efficient outcomes when leading firms acquire brands. If brand creation and brand development are simply activities that poach customer capital from other firms, the costs of concentration are higher than we find here. In this paper, we work in line with the significant empirical literature that points to considerable consumer benefits from new products and products with higher brand appeal, but we believe these questions will be important in further research on the macroeconomic implications of brands.

Figure 8: Effects of Reallocation Tax/Entry Subsidy on BGP



(a) Reallocation Tax

(b) Entry Subsidy

Notes: Welfare response to taxes and subsidy outcomes. Source: Author calculations.

6.4 Policy Analysis

This section explores various policy tools that may be implemented in markets where policymakers are either interested in the costs of market concentration or the benefits of innovation. We apply some standard policy tools (e.g., transaction tax or entry subsidy) and ask about the economic implications of these policies for concentration, growth, and welfare. Our framework provides a general equilibrium setting where both growth and concentration of the market are endogenously determined by firms' innovation and reallocation activities, and have welfare consequences.

Reallocation Taxes. Figure 8 focuses on the effects of reallocation taxes (panel a) and entry subsidies (panel b) on welfare along the balanced growth path in the homogeneous group estimation. Welfare responds positively to entry subsidies since these subsidies both alleviate the distortion from concentration and increase growth.

Table 10 focuses on the effects of different taxes and subsidies on reallocation. We perform these policies in one world where all markets have the same structure in panel (a), and one world where we allow the parameters to vary by group in panel (b). In these counterfactuals, we study the response to policy in terms of aggregate concentration (leader share), growth rate, and welfare (in both the steady-state BGP and the transitional dynamics).

First, we note that taxes on reallocation have the expected effects on the leader's market share. Against a baseline of 33% market share, taxes on reallocation can reduce the leader's steady-state share

Table 10: Counterfactual — Reallocation Tax and Entry Subsidy

	10% Reallocation Tax	10% Entry Subsidy
a. Homogeneous Groups		
Leader's Market Share (p.p)	24.21	21.02
Δ Growth rate (p.p)	-0.012	0.259
Δ Welfare (BGP, p.p)	-0.432	5.84
Δ Welfare (Transition, p.p)	-0.511	5.23
a. Heterogeneous Groups		
Leader's Market Share (p.p)	26.94	21.56
Δ Growth rate (p.p)	-0.009	0.022
Δ Welfare (BGP, p.p)	-0.293	4.85
Δ Welfare (Transition, p.p)	-0.133	4.21

Notes: Reallocation tax and entry subsidies, outcomes in counterfactual. Source: Author calculations.

significantly, to 24%. However, the net welfare effects are negative, since the overall growth rate also exhibits a significant negative decline. This growth decline occurs through both the static loss from reallocation of brands to better firms and the dynamic loss from entry. Completely shutting down reallocation is a costly policy.

The efficiency gains (in aggregate appeal) from reallocation overall outweigh the strategic losses (from pricing distortions). This implies that antitrust policies such as taxing transactions may not be efficient, if done at the *aggregate*. However, when we look across groups, there may be a set of groups where taxing transactions would be efficient. Thus, a key question for policy is at what level it is implemented. A coarser policy that does not take into account the rich market dynamics of each sub-market may induce efficiency losses.

Entry Subsidies. Figure 8 and Table 10 also present the effects of subsidies on entry. As in the previous table, we perform these policies in one world where all markets have the same structure, panel (a), and one world where we allow the parameters to vary by group, panel (b). In these counterfactuals, we study the response to policy in terms of aggregate concentration (leader share), growth rate, and welfare (in both the steady-state BGP and including the transitional dynamics).

We find a strong effect of entry subsidies on both concentration and growth. Since fringe firms have an easier time engaging in brand creation than incumbents, the subsidy induces a lot of fringe firm entry. This also increases growth and welfare significantly. Subsidizing entry is a better means of reducing market concentration and increasing growth than focusing on taxes or blocking reallocation.[22] Large firms engage in less brand creation than fringe firms, and this allows for within-fringe reallocation and declines in inefficient concentration. As in standard growth models, the entry margin is on net quite strong. This result also parallels work finding underinvestment in R&D and new product subsidies an optimal policy from Jones and Williams (2000), with the additional point that entry subsidies will shift

[22]This result is also budget-balanced, so this is true even though subsidies induce higher government expense than taxes.

the firm composition and reduce markup distortions through this channel.

This connects to recent discussions on rising regulatory compliance costs and uncertainty (Davis, 2017), which are likely harder for entrants to manage than incumbents. According to our framework, if these are read as "taxes" on entry, they can have significant welfare effects through both higher concentration and lower growth.[23]

6.5 Discussion

This section discusses two aspects of the quantitative framework that deserve some additional emphasis in their relevance for counterfactual analysis.

The Role of Maturity. The downstream innovation response to brand reallocation is a function of the interaction between brand maturity and reallocation. As a result, to a first order approximation, policymakers can ignore the innovation effects of antitrust policy *when transactions are of mature brands*, because the discounted value of transactions to entering firms is low. However, there is a rising tendency for brands to exhibit shorter life cycles and become transacted earlier in their life cycle. For transactions early in the brand life cycle, the dynamic effects of reallocation become more relevant, as the option value of selling for an entering fringe firm becomes more relevant.

We discuss these results quantitatively here. Recall ι measures the speed of brand maturity. We evaluate the policy of shutting down reallocation with three benchmarks in Table 11.

Table 11: Counterfactual — Maturity and Efficiency w/ Shutting Down Reallocation

	Baseline	Fast Maturity ($\iota \times 10$)	Slow Maturity ($\iota/10$)
Change in Leader's Market Share (p.p)	-11.11	-9.23	-17.29
Change in Growth rate (p.p) (%)	-0.321	-0.982	-0.141
Welfare (BGP, p.p)	-1.930	-21.75	-0.023
Welfare (Transition, p.p)	-1.332	-19.36	-0.001

Notes: Three counterfactual maturity scenarios. Source: Author calculations.

From column (1) to column (3), we consider how a different maturity rate of brands (with $\iota = 4\%$ as the estimated baseline) leads to different market concentration and welfare incidence. We add two extreme cases in columns (1) and (3), one where a brand grows at an average of 0.4% per year until peak, and another where a brand grows at an average of 40% per year until peak. We then compare changes in the innovation cost for entrants (κ_e) and changes in the search cost for brand reallocation (κ_s, as a stand-in for an ownership transaction tax). The results are striking, and suggest the maturity channel cannot be ignored in innovation and antitrust policies.

[23] We discuss in some more detail the robustness of our main results to theoretical and empirical specifications in Appendix E.

When markets mature quickly (e.g. average growth of 40% to peak), there is a large growth and welfare cost to shutting down reallocation (22% welfare cost). This is because the policy has a larger effect on both entry and reallocation. When maturity is slower (e.g., average growth of 0.4% to peak), shutting down trade decreases the leader's market share with a minimal impact on welfare (close to 0%). This occurs because the decline in reallocation has very little effect on entry, but significantly reduces concentration.

As for policy recommendations, both across industries and over time policymakers need to understand the life cycle profile and the age distribution of transactions. If older brands are much more likely to be sold, the focus on transactions will weigh the markup and efficiency effects. The framework in our model can still be used to link market shares, efficiency, and markups.

Yet, in markets where young brands are reallocated, policymakers should note the interaction between entry and reallocation. Entry responds positively to reallocation, in particular if reallocation rates are linked to young brands (see Table 8). If policymakers focus only on the predictions on sales and prices, they may miss the dynamic effects and induce efficiency losses by simply looking at the problem in a static setting.

The Demand System. Our demand system with nested CES structure follows a host of papers that study product markets, and we think this framework properly captures the competition for product market share. One might wonder, what is the effect of changing the demand system to a different type of system (e.g. an aggregator as in Kimball, 1995)? Our results qualitatively go through. The key distinguishing aspect is that, with a Kimball aggregator, moving brands from fringe to leading firms (our context, following Atkeson and Burstein, 2008) is similar to moving brands from small to large firms (Kimball, 1995). Both Kimball (1995) and Atkeson and Burstein (2008) generate pricing power gaps that emerge through market concentration. We believe that nested CES is the most appropriate demand system to discuss the role of multi-product firms and the mechanisms through which firms build their market power through brand creation and reallocation. This is a demand system where the role of brand appeal is natural.

7 Conclusion

Brand capital is a central component of the modern economy, and brand reallocation plays a major role in sales concentration, firm dynamics, and efficiency. We employ a novel dataset on the universe of brands to unpack the role of brand reallocation and brand dynamics in the macroeconomy. Empirically, we find that brand creation plays a much larger role for small firms than for large firms, while brand reallocation plays a major role in determining large firms' market shares. For both, the life cycle of the brands they hold is a crucial component of their market shares.

To understand the efficiency implications of these forces, we introduce a model of multi-product firms that innovate and acquire brands with productive and strategic incentives. In our quantified model, large firms tend to be more efficient than smaller firms but have more pricing power through amassing brand capital. This efficiency-markup tradeoff leads to a natural tension between economic growth and market concentration, which we study with the estimated model. We estimate the model using our detailed data to study a set of relevant policy counterfactuals: how does shutting down or taxing brand reallocation affect consumer welfare and efficiency? How does subsidizing brand entry affect these outcomes? How do these policies interact with the brand life cycle?

We find taxes and blocking reallocation to large firms tend to reduce concentration and growth, leading to lower welfare. Subsidizing brand entry is a policy that can target the same concentration level with a positive impact on growth. Further, there is significant heterogeneity across product groups. If policy is coarse, taxes and subsidies on reallocation may decrease economic efficiency. If policy can be applied by group, there may be gains from subsidizing reallocation in some groups and taxing reallocation in others. However, for the same level of concentration, brand entry subsidies persistently appear to induce more growth and be more welfare-enhancing.

Empirically, one avenue for further research is to understand the long-run evolution of the market for brands and long-run changes in ownership structure. These findings would touch on important topical economic questions in innovation, concentration, and the role of intangible assets in firm dynamics. Understanding the brand-firm interaction is essential to understanding the trends in market shares and market dynamics. Theoretically, as the importance of brand capital continues to rise, frameworks that address the connection between brands, concentration, and growth will be essential for academic and policy discussions. We expect to see brands playing an important role in linking firm dynamics to market shares and the aggregate economy. Placing brands into an endogenous growth framework provides a new foundation for understanding the joint determinants of markups, concentration, and growth.

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Appendix

Our Appendix is in five sections, mirroring the structure of the text. Appendix A discusses the data background and general points about large firms and brand acquisitions. Appendix B discusses the empirical analysis connections to the literature and robustness. Appendix C discusses the theoretical proofs and expands on the firm's dynamic problem. Appendix D discusses the estimation. Appendix E discusses the general robustness of the quantitative results. For an updated Online Appendix discussing technical details, please see liangjiewu.com/files/tm_pw_apx_oct22.pdf.

A Data Appendix

This section addresses the set of data sources relevant for the analysis and the data examples that motivate our investigation. Section A.1 motivates the general setting by evaluating examples of market concentration and brand building at the firm level. Section A.2 expands on the details of the merge across datasets.

A.1 Product Market Concentration: Large Firms

A salient feature of markets for intellectual property is the common rate of exchange, in particular with multi-product firms. We find this in both our data sources (USPTO and RMS Nielsen), and additionally from investigating firms in company reports of acquisition. Firms detail their major acquisitions and reasons for acquisitions in press releases. Firms claim different reasons for acquisition. Often the motivating claims for acquisition are close to the two main theoretical mechanisms in the paper, with many acquisitions claiming often synergies or good product-firm fit (e.g., with the General Mills purchase of Annie's in 2014)[24] and others focusing on the importance of market leadership (e.g., Nestle acquiring Dreyer in 2006).[25]

These persistent transactions lead to the observed skewed distribution of firms, and the fact that many firms hold brands they did not originally introduce. Both of these forces can be seen in Figure A1, where many brands that individuals associate with only the brand are held by larger parent firms.

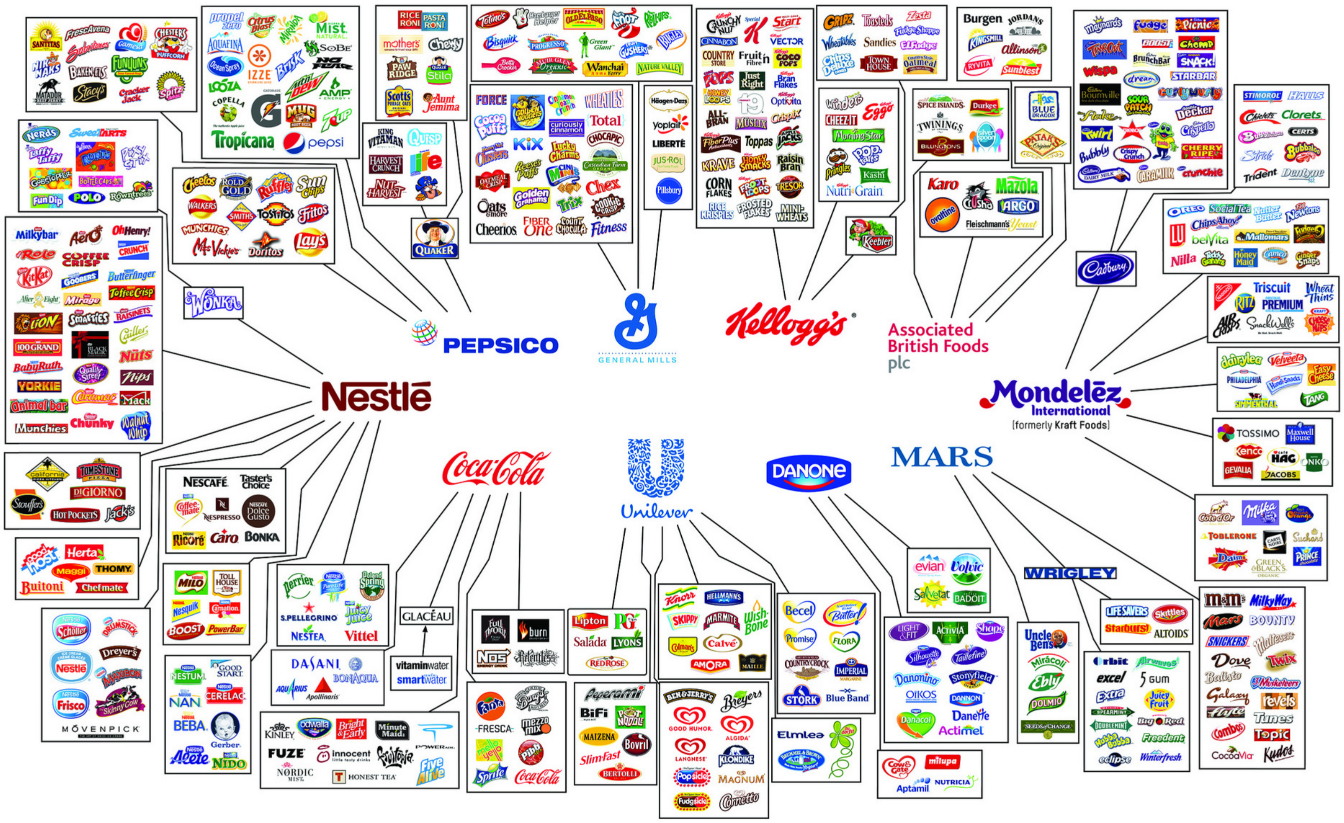
Figure A1 illustrates how many distinct brands are owned by the same firm. In Figure A1, for example, around half of the brands originally started at a different firm from the one it is currently linked to. Further, most brands are mature and took some time to build customer capital. Both findings complement the key ingredients of brand maturity and reallocation in our framework.

This general pattern is true across an array of industries, but the empirical section of this paper directs our attention to the Consumer Packaged Goods (CPG) industry, with the limited consumer substitution.

[24]Source:https://investors.generalmills.com/press-releases/press-release-details/2014/General-Mills-To-Acquire-Annies/default.aspx

[25]Source:https://www.nestle.com/media/pressreleases/allpressreleases/dreyersandworldleadericecream-19jan06

Figure A1: Brands at Major Firms



Notes: Parent Companies for a given brand. Source: The Independent, 2017.[a]

[a] https://www.independent.co.uk/life-style/companies-control-everything-you-buy-kelloggs-nestle-unilever html, Apr 2017, accessed September 2022

We further illustrate this force by showing the progressive increase of brands in Procter and Gamble (P&G) and Johnson & Johnson (J&J), two large companies that hold many brands. We see that their stock of live trademarks is increasing over time, and the hundreds of trademarks seen in Figure A2 represent a host of brands.

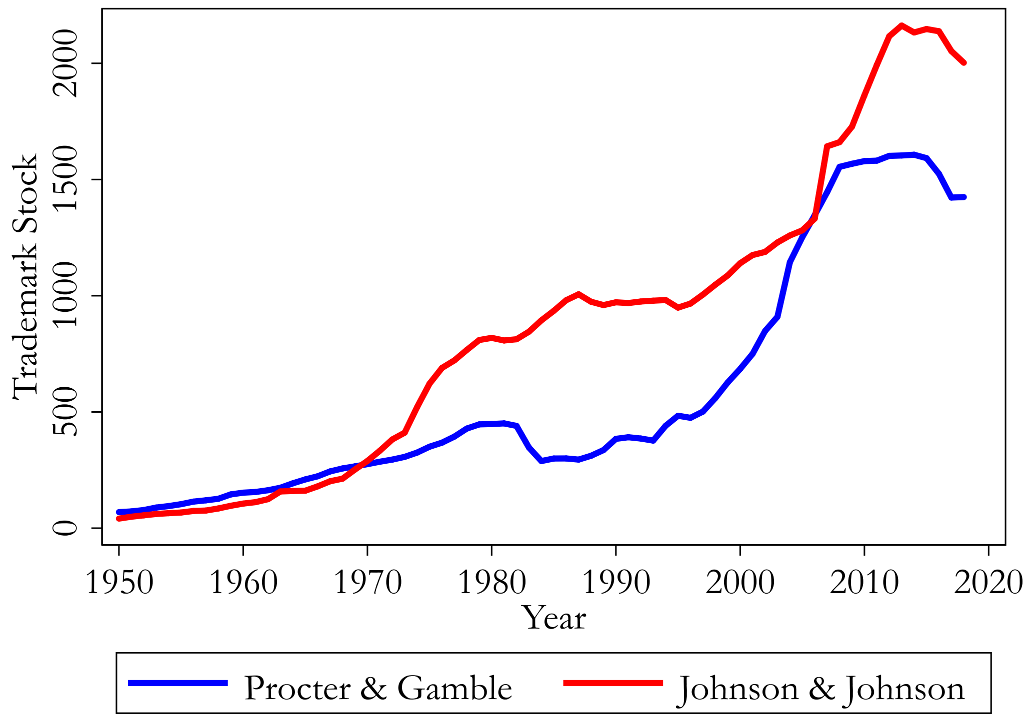
A.2 Data Merge Details

As discussed previously, our main merge links USPTO Trademark data with RMS Nielsen Scanner data. We proceed by linking firms and products separately. Our merge matches over 80% of sales-weighted products. Some problems still emerge with short-names. We use "tokens" and fuzzy matches to deal with the names. Firms and products follow similar procedures and we discuss them in turn.

Firms. For matching firms, we first standardize on a large set of firm tags, eliminating common firm words, e.g. "CORP", "INC", "ESTABLISHMENT").[26] We then take the cleaned and standardized name

[26] The full list is here ('AB', 'AG', 'BV', 'CENTER', 'CO', 'COMPANY', 'COMPANIES', 'CORP', 'CORPORATION', 'DIV', 'GMBH', 'GROUP', 'INC', 'INCORPORATED', 'KG', 'LC', 'LIMITED', 'LIMITEDPARTNERSHIP',

Figure A2: The brands of P&G and J&J over time



Notes: This collects the total stock of trademarks held by P&G and J&J in each year since 1950. Includes trademarks held through registration and assignment. Source: USPTO.

and match according to a tokenized bigram matching procedure.

Brands. By focusing on brands, we direct our attention to long-running products held by firms. USPTO Trademark data provides the "tm_name" or the name associated with a registered trademark. RMS Nielsen follows a similar format, which has a "brand_name". We join the two by employing a token name matching. For brand names, there are no further removals of tokens beyond the firm-level analysis.[27] For brand age, we focus on the "prior" brand, as in the broader brand umbrella of the production. For transacted brands, we observe the level of the transaction and focus on this.

Transactions. As we discussed previously, we leverage evidence from transactions in both USPTO and RMS Nielsen Scanner data. Overall, we get 20% of brand transactions from USPTO Trademark data and 80% of transactions from Nielsen. While there are more transactions observed in trademark data, there are some within firm transactions we drop, as we generate a text similarity threshold above which we do not consider transactions.

'LLC', 'LP', 'LTD', 'NV', 'PLC', 'SA', 'SARL', 'SNC', 'SPA', 'SRL', 'TRUST', 'USA', 'KABUSHIKI', 'KAISHA', 'AKTIENGESELLSCHAFT', 'AKTIEBOLAG', 'SE', 'CORPORATIN', 'GROUP', 'GRP', 'HLDGS', 'HOLDINGS', 'COMM', 'INDS', 'HLDG', 'TECH', 'GAISHA', 'AMERICA', 'AMERICAN', 'NORTH', 'OPERATIONS', 'OPERATION', 'DIVISION', 'COMPAGNIE','INTERNATIONAL', 'NORTH AMERICA', 'InBev').

[27]Standardizations include removing any relevant firm names as discussed in the firms section, but does not do any further standardizations and tracks the token grams within each brand name.

B Empirical Appendix

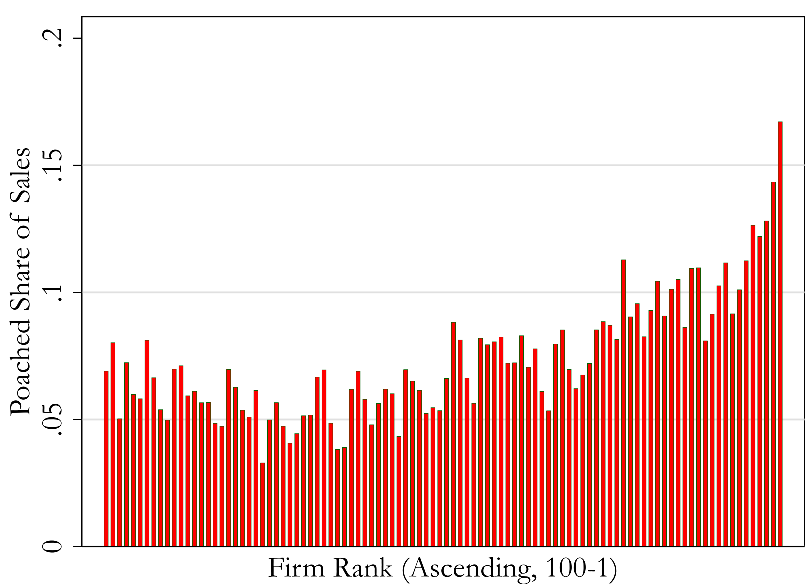
This section explores some additional evidence on a couple core messages from the paper, focusing in particular on the firm, brand, and firm $\times$ brand analysis. We apply broader data from the USPTO to indicate the fact that large firms build large portfolios of brands and their acquired brands drive a larger share of their portfolio. We then discuss the product life cycle with reference to the literature and discuss the integration of the product life cycle with our firm-level analysis. In each case, we explore the robustness of our results to varying definitions.

We start by expanding on the main elements of firm analysis in Section B.1, returning to the study of the sources of concentration, and evaluate the robustness of the empirical results on firms. We expand on the product life cycle in Section B.2, focusing on the interaction of age and sales, and the evidence for the importance of product maturity and sales dispersion over time. We further discuss our connection to the literature on the product life cycle and then turn to the robustness of product-level results. We then explore the event studies and the interaction of reallocation flows across firms in Section B.3. Lastly, we discuss the types of reassignment in the trademark data in Section B.4, which is in part a plea for further research to investigate further the sources and implications of IP reallocation.

B.1 Firm-Level Analysis

In Figure 1, we showed how buying of brands contributes significantly to large firms market share. Figure B3 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the percentile (running from 1-100) of the firm size in sales.

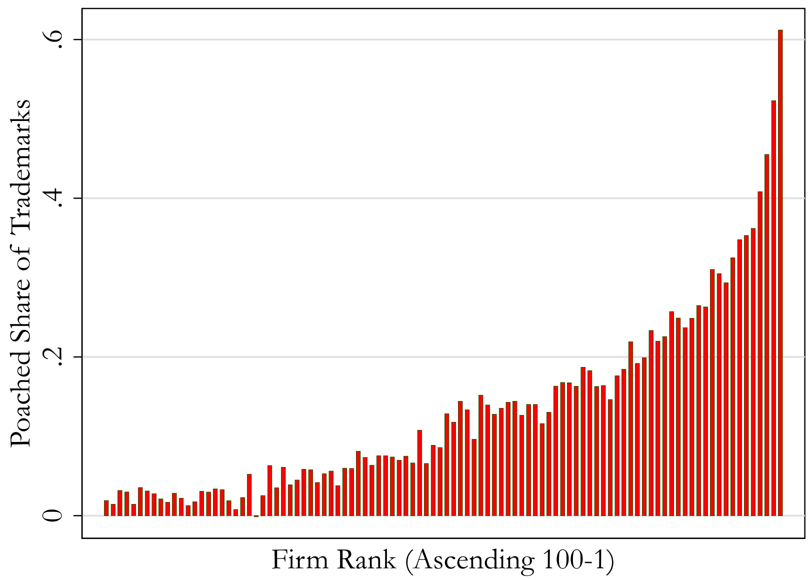
Figure B3: Contribution of Buying to Sales Share



Notes: Share of total sales from poached brands. Source: RMS Nielsen and USPTO Trademark

We find that the highest-selling firms have almost 4-times as much poached share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data. We observe this in both RMS Nielsen Scanner data and in USPTO Trademark data. Turning to USPTO data, we find the results are even more stark. Large firms tend to carry bought trademarks as a much larger share of their portfolio. This is noted in Kost et al. (2019), and can be seen in Figure B4.

Figure B4: Contribution of Buying to Trademark Stock



Notes: Market share reallocation measures across different firm types, following Equation (1). Source: RMS Nielsen

In Table 3, we explored the concentration due to incumbent products versus entering or reallocated products. However, incumbent variation may be driven by many forces outside of the life cycle of the product. Here, we combine our life cycle analysis with the variance decomposition in Equation (1).

we run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{37}$$

Equation (37) focuses on three different margins for y_{it}, with the entry and reallocation following the same structure as Table 3. We substitute y_{it} as the fitted value of sales on maturity ("Fitted Maturity") to understand how much of the observed variation from maturity is due to predictable life cycle growth. Table B1 evaluates the contribution of each force in the equation.

We find similar patterns in Table 3 and Table B1. When we take fitted values from product-level age regressions, as discussed in the next section, the general pattern stays the same. We note that each force has a non-negligible contribution to the distribution of market shares, and our empirical model explains around 85% of the variation for large firms, and 70% of the variation for fringe firms.

Table B1: Sources of Reallocation

	(1) Entry	(2) Fitted Maturity	(3) Reallocation	(4) Unexplained
Leader	0.033*	0.70*	0.13*	0.14
Fringe	0.091*	0.57*	0.021*	0.32

* $p < 0.001$

Note: Market share reallocation measures across different firm types, following Equation (37). Source: RMS Nielsen.

B.1.1 Empirical Robustness: Firm Measures

In our robustness, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary owner of a brand. This robustness section focuses on the empirical facts at the firm level addressing some changes to these definitions.

The results in Section 3.1 delivered two main messages. First, markets are highly concentrated, as the largest firm is more than 1000-times larger than the median firm across most product groups. This is noted in Figure 1. Second, large firms' outcomes are more driven by brand maturity and brand reallocation, while smaller firms rely much more on brand entry. This is noted in Table 3 and Table B1.

This current section explores varying the definitions of the firm-product relationship. We explore the differences in both the RMS Nielsen data on its own and USPTO Trademark data. When exploring RMS Nielsen data, we expand our sales to include unmerged brands and brands with sales less than $1000. As a result, we revisit Section 3.1 with different definitions of firm ownership. In overview, the results are qualitatively similar. We turn to the two main departures in our definition of the firm. We first look exclusively to Nielsen scanner data and then USPTO Trademark data.

RMS Nielsen Scanner. Our first fact primarily employed RMS Nielsen Scanner data, but we only included the successfully merged products to maintain a consistent sample. Given the success of the merge, one should expect the general results to be similar. In this section, we confirm that intuition.

The average top firm share is 32% of the total market in the main part of the manuscript. When we expand our set, we find that the results are similar. We visit the shares in Table B2, as below, where we drop any external observed reassignments:

Table B2: Firm Market Shares in 2010, Restrict to Merged w/o Adjustment

Top firm share by group	Top 2 firm share	median share
30.4%	45.4%	0.01%

Without adjusting the weights by the overall sales of a group, we find a similar skewness in firm size albeit with the top 2 firms having a larger share, as well as the median firm. This can be seen in Table B3.

Table B3: Firm Market Shares, unadjusted weights

Top firm share by group	Top 2 firm share	median share
33.6%	50.3%	0.06%

USPTO Trademarks. We now define firms at the USPTO level rather than the Nielsen level to explore different patterns in share holdings. For unidentified transfers[28], we maintain the originator as the parent company.

B.2 Brand-Level Analysis

In this section, we expand on the brand-level discussion in the main text, referring to brands and products interchangeably unless specifically indicated. Products are both a significant source of firm concentration (Hottman et al., 2016), yet highly dynamic (Argente et al., 2020a). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The change in products can come from development of a product line or transactions of products from worse to better firms. Our goal in this section is to isolate the product element of the life cycle and show how even separate from the firms that hold them, products exhibit rich life cycles. This general point has been shown before (e.g. Argente et al., 2020a, 2021), but by integrating with USPTO Trademark data we are able to examine the longer brand life cycle and control for the transactions across firms.

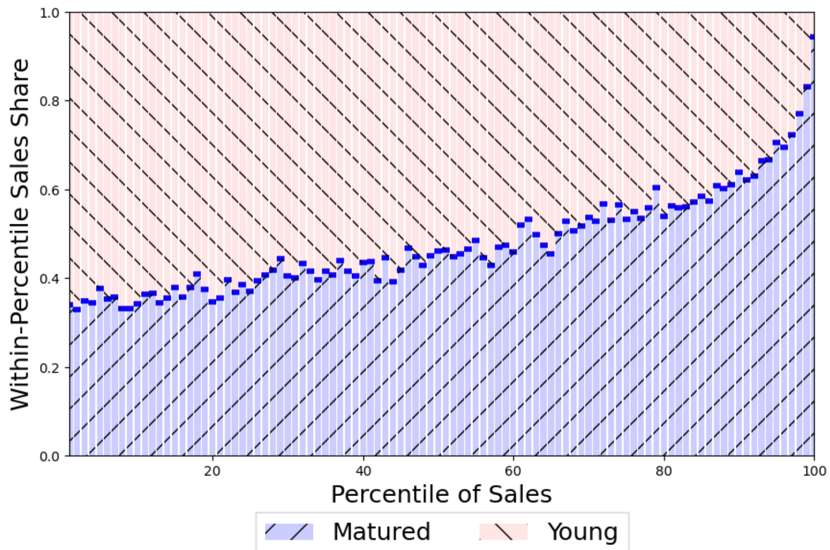
Some products charge to dominance quickly, others rise gradually but maintain leadership, whereas others survive but remain in obscurity. Yet all brands must build customer capital to build market share. We direct our attention to brand *age* as a key ingredient to product market shares. We first focus on a snapshot of the distribution of sales by age, and then turn to an analysis of the life cycle to understand the more granular dynamics.

Products evolve over their life cycle. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure B5 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[29]

For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

[28]There are cases where trademarks are reallocated to unidentified firms, and we limit our use of these observations.
[29]We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

Figure B5: Brand Percentile and Maturity



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young").

Source: RMS Nielsen Scanner Data.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high customer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age. This is noted in the main text in Equation (2) and Figure 2a. Yet, we are not the first to focus on this life cycle so we review the current literature benchmarks here.

Literature Benchmark: The Product Life Cycle. As discussed in the main text, our findings on the brand life cycle are significantly longer than the life cycle discussed in recent work (e.g. Argente et al., 2018). Here, we crosswalk our results to existing work on the product life cycle to benchmark where we diverge. Argente et al. (2018) focus on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to the main text and compare them to a relevant current paper in the literature, in particular focusing on defining age in two different ways, to ensure the differences in the age profile does not simply come from applying a dataset with different age measures. Equation (38) presents the regression:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{38}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table B4 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Table B4: Log Sales, by Nielsen and Trademark Age

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00)	(0.00)	(0.00)	(0.00)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Notes: Balanced Panel Life Cycle Regressions of Log Sales on Age, utilizing different age sources and different variation.
Source: USPTO Trademark and RMS Nielsen

Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a significantly longer brand life cycle than found in Argente et al. (2018). We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level.

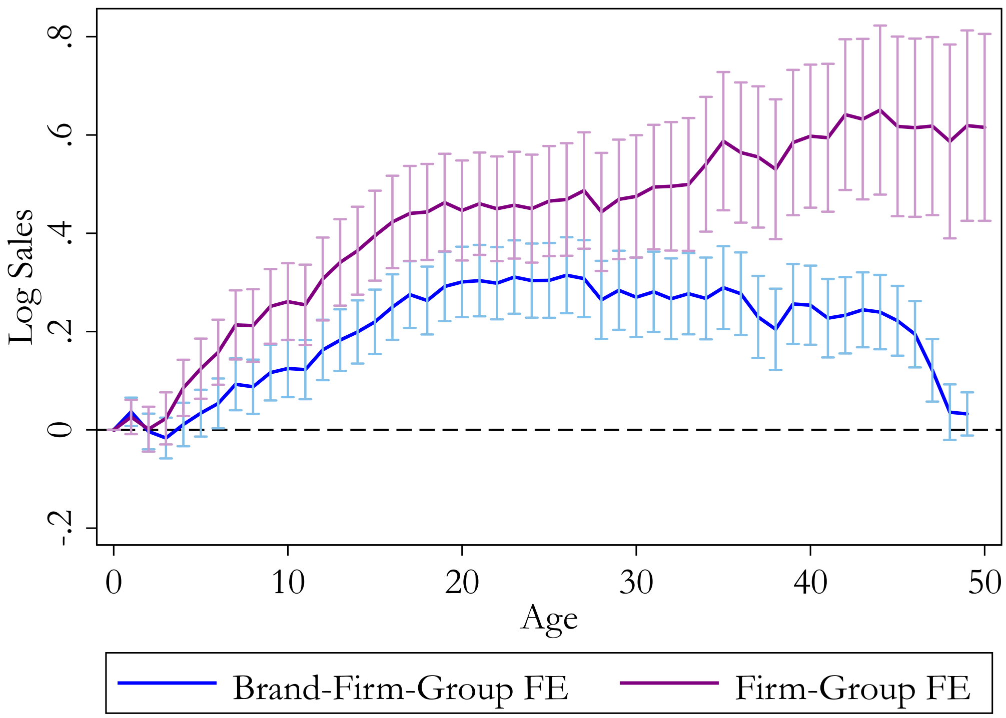
Figure B7 evaluates the life cycle profile within a given product group code. We follow the regression in the main text, except in prices we weight by sales share. Equation (39) illustrates the structure of the regression.

$$\log y_{ijkt} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_{ikj(i)} + \epsilon_{ijkt} \tag{39}$$

The regression in Equation (39) considers the sales and prices of brand i with firm j in group k at time t, $\log y_{ijkt}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[30] The $\theta_{ikj(i)}$ indicates a brand-group or firm-group fixed-effect. Figure B7 plots the regressions by age coefficient β_a.

[30]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

Figure B6: Life Cycle Regressions



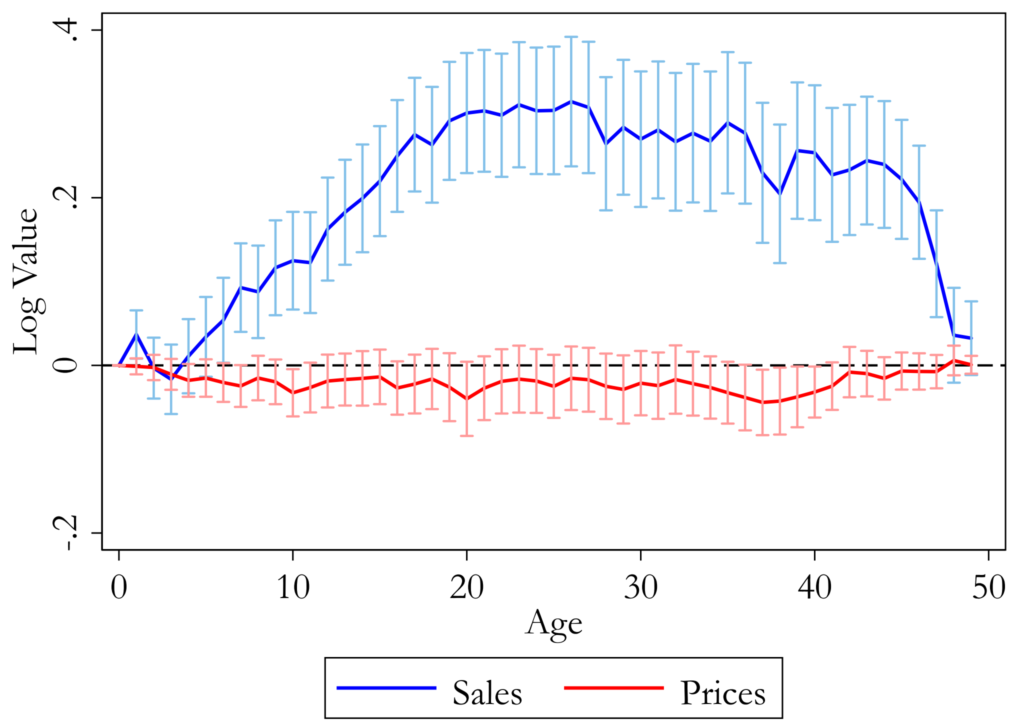
Note: Plots of log sales on age regression coefficients, controlling for brand-group and controlling only for firm-group. 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

Figure B7 is consistent with the main facts from Section 3.2. We note that the inverted-U profile is still persistent within group, though with a slightly lower peak than in the brand's overall life cycle. We also note that the life cycle of prices shows on average somewhat minimal activity for the brand across age. This means that the strategic pricing firms engage in does not appear to be correlated with age, though as we have noted from events there are shifts in prices, consistent with previous evidence in the literature.

Coarsened Exact Match: Details. In this section, we expand on the coarsened exact match procedure in Section 3.3, discussing the method we use to link brands to their counterfactual brands prior to the event. We link brands that are reallocated to matched brands that have the same change in log sales in the previous two periods to the event period of the reallocated brand, the same year, and the same age bin (where we define age bin in 4 groups: 0-7, 8-19, 20-36, 37 or older). We perform the weighting following Blackwell et al. (2009), and take a synthetic control that we compare with the treated brands.

Definitions. In this section, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary

Figure B7: Life Cycle Regressions, Prices and Sales within Group



Note: Plots of log sales and prices on age regression coefficients, controlling for brand-group, as in Equation (39). 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

owner of a brand. This robustness section focuses on the empirical facts at the firm level addressing some changes to these definitions.

Product Definition. We focus on the product life cycle in our data, but aggregate across all brands in the main maturity specification to avoid brand $\times$ product group features. The life cycle peaks around the same time in both specifications (see the peak age in Figure 2a and Figure B7). However, when we analyze brand $\times$ group, the life cycle peaks at a slightly younger level (0.35 versus 0.45). This should not change the qualitative implications of our results.

Transaction Definition. Transactions are defined at both the Nielsen and USPTO level. The reason we define transactions using both is as follows. We note that when we plot the results applying only USPTO transaction information we find as follows. Multiple serial numbers per brand.

B.3 Empirical Robustness: Firm $\times$ Brand Analysis

In the main paper, we focused on the responsiveness of sales and prices to both events and allocation to top firms. Here, we discuss different definitions of top firms and events to understand the general robustness of our results. We find qualitatively very similar results, which would not change the main messages of our analysis.

Prices and Sales at Top Firms. In this section, we explore varying the definition of a top firm to understand the differences in predicted sales. Table B5 focuses on the robustness of the higher log sales at larger firms. We see that larger firms tend to show higher sales of the same brand.

Table B5: Log Sales Conditional on Holding Firm, Trademark Age Fixed Effects

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales	(5) Log Sales	(6) Log Sales
Top 10 Overall	0.57***	0.55***				
	(0.000)	(0.000)				
Last Period Top 10			0.59***	0.69***		
			(0.000)	(0.000)		
Top 10 in 2006					0.47***	0.53***
					(0.000)	(0.000)
N	441300	3972	441300	3972	441300	3972
R^2	0.844	0.741	0.844	0.735	0.844	0.740
Weights	No	No	No	No	No	No
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table B6 focuses on the robustness of the higher log prices at larger firms, focusing only on the merged sample. We note that the results directionally hold, but exhibit a higher variace.

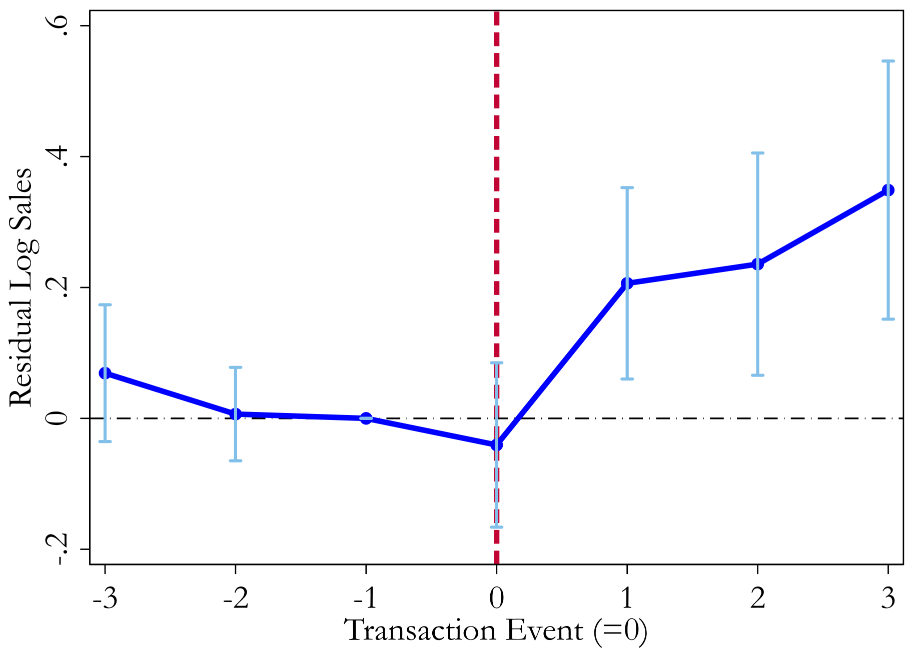
Table B6: Log Price Conditional on Holding Firm, TM age FE

	(1) Log Price	(2) Log Price	(3) Log Price	(4) Log Price	(5) Log Price	(6) Log Price	(7) Log Price	(8) Log Price
Top 10 Firm	0.33	0.26*			0.057	0.036		
	(0.177)	(0.062)			(0.170)	(0.403)		
Top 10 Firm in 2006			0.14	0.34*			-0.0091	0.029
			(0.350)	(0.088)			(0.869)	(0.516)
N	441300	3972	441300	3972	441300	3972	441300	3972
R^2	0.967	0.881	0.967	0.882	0.983	0.983	0.983	0.983
Weights	Total Wt.	Total Wt.	Total Wt.	Total Wt.	Period Wt.	Period Wt.	Period Wt.	Period Wt.
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Figure B8: Coarsened Exact Match and Brand Transaction Leader-to-Fringe, Log Sales



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

Gross Flows and Net Flows. One of the main aspects of our paper focuses on the reallocation of products across firms. We identify this reallocation by jointly using RMS Nielsen Scanner data and USPTO Trademark data.

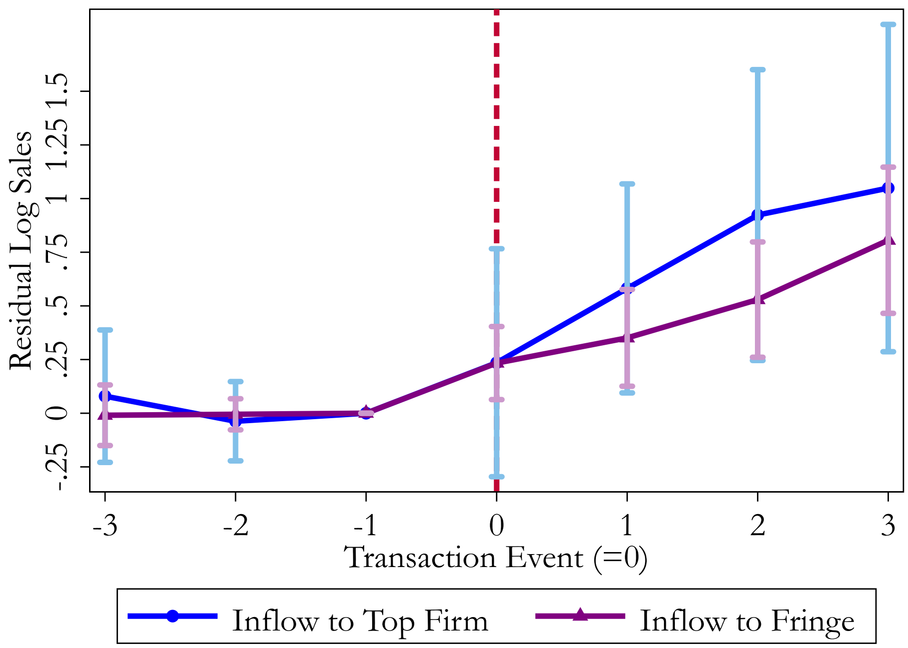
Event Studies. Our event studies focus on transactions across firms in the data. For an observed transaction, both the buyer and the seller must exist in the data. We employ a balanced panel with seven periods. Given we use data from 2006–2018, we must restrict our event study analysis to brand transactions from 2009–2015. Due to some of the restrictions on our data, we focus on a broader definition of leading firms and flows from low-type to high-type firms. We explore the robustness of event studies depending on our characterization of an event study and definition of firm type.

To characterize flows that link fringe and leader buyers and sellers, we evaluate exchanges that move from smaller sellers to larger buyers, defined over the horizon of the sample. We make a couple of adjustments to the definition of a large firm to evaluate the robustness of our event study results.

Lastly, we compare more broadly the change in prices and sales upon the inflow of a brand to a large and small firm. We consider a large firm to be a top 10 firm within the product group code, and a small firm to be all other firms. We ask how prices and sales respond by doing the same analysis here. Limiting attention only to brands that move between firms, we also evaluate the price and sales differences depending on the holding firm in Table B6.

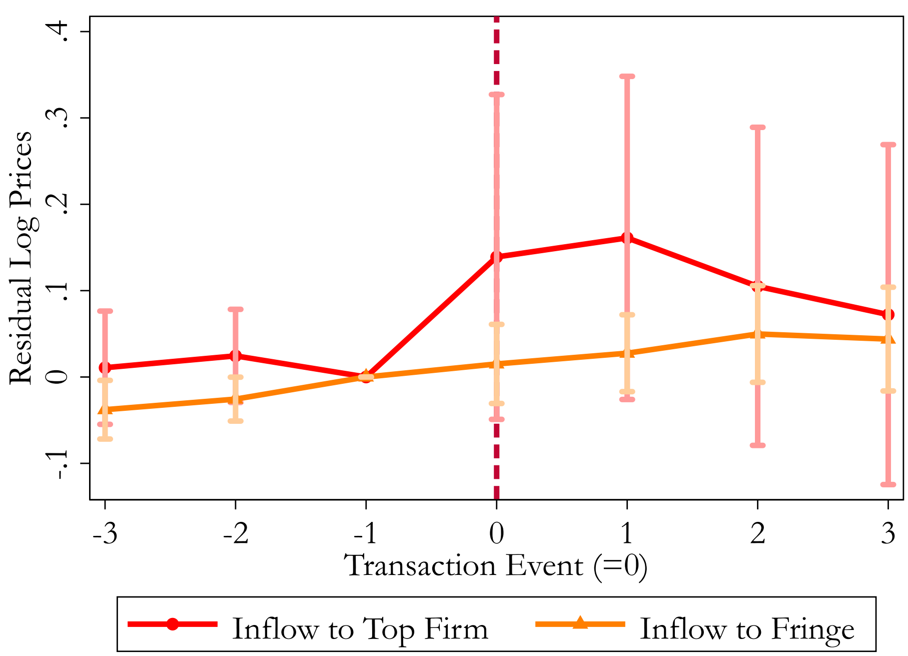
Figure B11 focuses on the different brand creation rates (entry as share of overall firm sales), and we

Figure B9: Inflow to Top Firm and Fringe, Sales



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

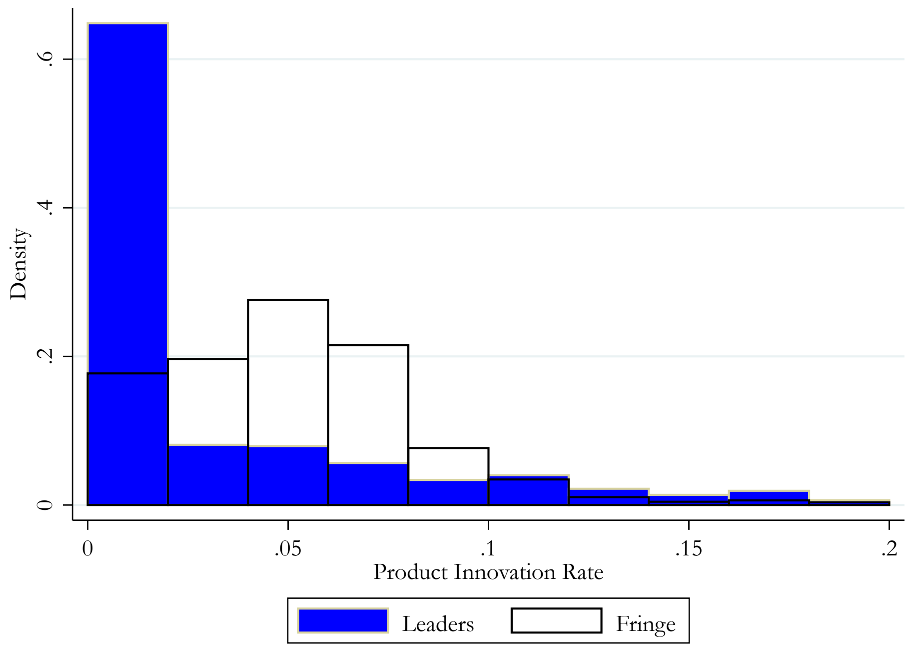
Figure B10: Inflow to Top Firm and Fringe, Prices



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

note the much stronger entry rate of fringe firms than leaders.

Figure B11: Brand Creation by Type



Notes: This looks at the brand creation distribution by firm type, *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader defined as in text.

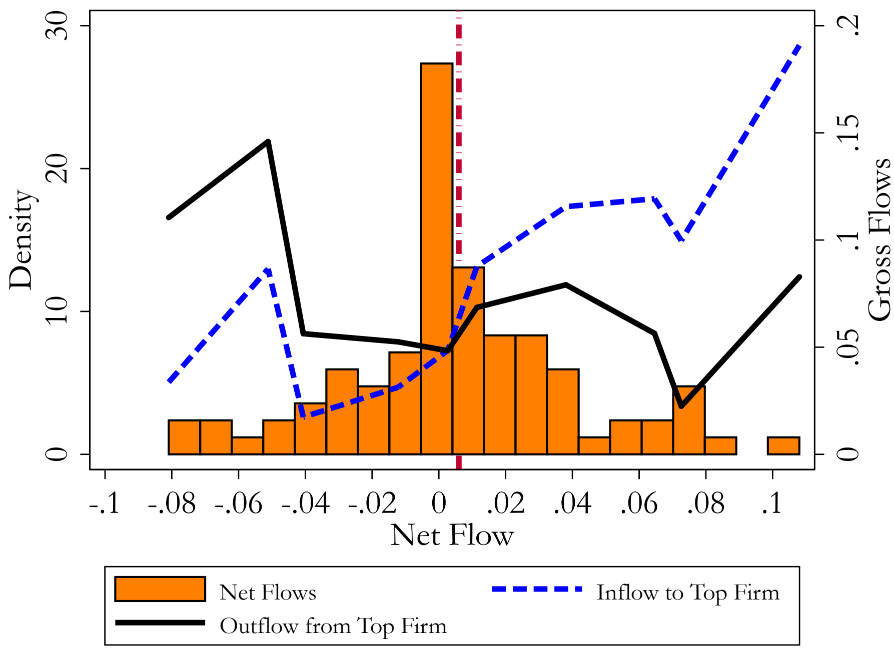
Further, we note that overall in the transfer of brand ownership there are more flows from small to large firms. This can be seen in firm press releases, as we observe many inflows and outflows of brand ownership for large firms, with inflows being more common. This can be seen in Figure B12, which collapsed the total brand flows (as share of sales) in both directions, with a histogram of net flows and line plots of gross flows.

B.4 USPTO Trademarks: Reassignment

The most reliable long-term data source for brand reallocation is USPTO Trademark data. Our focus in this paper is particularly on reallocation due to either pure reassignment (e.g. ownership transfer) or mergers & acquisitions. In this section, we discuss the general contours of the trademark data when it comes to reallocation of ownership. There is significant reallocation in the data, but some reallocation does not fall under the specific "merger" or "reassignment", but instead is linked to name changing, collateral, and other corrections and adjustments.

Table B7 splits the different transactions in the data into their different groupings. Most transactions in the data are available from 1970-2018. We order the transaction type by largest share of transactions. However, each transaction may contain a bundle of trademarks (e.g., transfer of ownership of "Odwalla"

Figure B12: Gross and Net Flows, Fringe and Leader



Notes: This looks at the market shares transferred across firms in market shares by year, averaged by product group code. *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader (top 10 firm) defined as in text.

may be bundled with various sub-brands of the core brand Odwalla). For example, in the case of "Security Interest" (or collateral), note that on average a larger number of brands are involved in the pledged bundle.

Table B7: Summary Statistics on Trademarks from USPTO

	Transaction Count	Trademark (TM) Count	TM/Transaction	Transaction Share	TM Share
Reassignment	478442	1.54M	3.21	**0.523**	0.345
Name Change	200767	795465	3.96	**0.219**	0.178
Security Interest	101280	1.10M	10.91	**0.111**	0.248
Merger	46610	287001	6.16	**0.051**	0.064
Correction	23500	119017	5.06	**0.026**	0.027
Other	64456	615334	9.55	**0.070**	0.138
Total	915055	4457996	4.87	1	1

Note: This table describes the category of each transaction in USPTO and orders them by their share of total transactions. Source: USPTO.

While our main focus in this paper has been mergers and reassignments, we note the richness of the data on multiple margins. Name changes are frequent, as firms may attempt to retool but maintain brand loyalty. Further, as noted previously, trademarks are often used as collateral. While Security Interest transactions are a small share of overall exchanges (around 10%), they make up almost 25% of all trademarks in exchanges. However, without transfer the firm may continue to operate these product lines. The benefit of focusing on mergers and reassignments is the reallocation of ownership and management across firms, but we hope to see further research on these margins.

C Theoretical Appendix

This section expands on some model discussion in the main text. Section C.1 expands on the leader's pricing decision in the main text. Section C.2 expands on the leader's dynamic problem in the main text, while Section C.3 expands on the equations and proofs in the main text.

C.1 Leader's Static Problem

Leaders attempt to maximize profits at each instant t. Recall the leader chooses prices subject to the demand curve as follows,

$$\max_{p_i} \int_{i\in\mathcal{I}_t^L} (p_i - e^{-z/(\sigma-1)}\mathbf{w}_t) c_t(p_i, \psi_i) di$$

s.t.

$$c_t(p, \psi) = \psi \times p^{-\sigma} \times P_t^{\sigma-1} \times \mathbf{C}$$

We include here the definition of the price index,

$$P_t = \left(\int_0^{N_t} \psi_{it} p_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \tag{40}$$

To simplify notation, we omit the time subscript. The first order condition w.r.t. price for each product i is:

$$0 = \quad c(p, \psi)\left[1 - \sigma(p_i - e^{-z/(\sigma-1)}\mathbf{w}_t)\frac{1}{p_i} + (\sigma-1)\int_{i\in\mathcal{I}_t^L} (p_j - e^{-z/(\sigma-1)}\mathbf{w}_t)\frac{\psi_j p_j^{-\sigma}}{\left(\int_0^N \psi_i p_i^{1-\sigma} di\right)} dj\right] \tag{41}$$

Using the definition of markups as price over marginal cost, $\mu_i = \frac{p_i}{e^{-z/(\sigma-1)}\mathbf{w}}$, we divide through by $c(p, \psi)$, and can re-write the first-order condition as:

$$0 = 1 - \sigma(1 - 1/\mu_i) + (\sigma-1)\int_{i\in\mathcal{I}_t^L} (1 - 1/\mu_j)\frac{\psi_j p_j^{1-\sigma}}{\int_0^{N_t} \psi_i p_i^{1-\sigma} di} dj \tag{42}$$

Guessing that $\mu = \mu_j = \mu_i$:

$$0 = 1 - \sigma(1 - 1/\mu) + (\sigma - 1)(1 - 1/\mu) \int_{i \in \mathcal{I}_t^L} \frac{\psi_j p_j^{1-\sigma}}{\int_0^N \psi_i p_i^{1-\sigma} di} dj \tag{43}$$

$$= 1 - \sigma(1 - 1/\mu) + (\sigma - 1)(1 - 1/\mu)s, \tag{44}$$

where the second equality comes from the definition of leader's market share. Inverting this equation we can write the markup of leader as a function of its market share:

$$\mu = \frac{\sigma(1-s) + s}{\sigma(1-s) + s - 1}.$$

Because the fringes can be viewed as a "leader" with zero market share, its markup is given by:

$$\bar{\mu} = \frac{\sigma}{\sigma - 1}.$$

Lastly in order to solve for the market share of the leader, we use its definition:

$$s = \frac{Q_L \mu^{1-\sigma}}{Q_L \mu^{1-\sigma} + Q_F \bar{\mu}^{1-\sigma}} = \frac{\phi \mu^{1-\sigma}}{\phi \mu^{1-\sigma} + \bar{\mu}^{1-\sigma}}$$

We discuss the proof of Lemma 1 in Appendix C.3 below.

C.2 Leader's Dynamic Problem

The leader chooses an innovation intensity (η), vacancies (o) and terms of trade (τ) to maximize the dynamic returns as follows,

$$\max_{\eta_t, o(\mathbf{x}), \tau^{LF}(\mathbf{x})} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t \right] dt, \tag{45}$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}},$$

$$B_t = \int \left[M(v_t(\mathbf{x}), u_t(\mathbf{x})) \tau^{FL}(\mathbf{x}) - o_t(\mathbf{x}) \right] d\mathbf{x},$$

$$S_t = \int \lambda(\theta_t(\mathbf{x})) \tau^{LF}(\mathbf{x}) n_t^L(\mathbf{x}) d\mathbf{x},$$

To characterize the optimal solution, we start by setting up the full Lagrangian:

$$
\begin{aligned}
&\mathcal{L}\Big(\eta_t, \nu_t(\mathbf{x}), \tau_t^{LF}(\mathbf{x}), v_t(\mathbf{x}), q_t(\mathbf{x}), \zeta_t\Big) \\
&= \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt + \int_0^\infty \zeta_t \left(\phi_t - \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}}\right) dt \\
&+ \int_0^\infty e^{-\rho t} v_t(\mathbf{x}) [\dot{n}_t^L(\mathbf{x}) - \underbrace{\eta_t f(\beta) \mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta) \frac{\partial n_t^L}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{LF}(\mathbf{x})\right) n_t^L(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})>0} f_\gamma(\gamma) \lambda\left(\theta_t^{FL}(\tilde{\mathbf{x}})\right) n_t^F(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^L(\mathbf{x})] dt \\
&+ \int_0^\infty e^{-\rho t} y_t(\mathbf{x}) [\dot{n}_t^L(\mathbf{x}) - \underbrace{\eta_t^F f(\beta) \mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta) \frac{\partial n_t^F}{\partial \beta}(\mathbf{Q})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{FL}(\mathbf{x})\right) n_t^F(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})<0} f_\gamma(\gamma) \lambda\left(\theta_t^{LF}(\tilde{\mathbf{x}})\right) n_t^L(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^F(\mathbf{x})] dt
\end{aligned}
$$

We rewrite the integral with $\dot{n}_t^L$ using integration by part:

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} v_t(\mathbf{x}) \dot{n}_t^L(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(v_\infty(\mathbf{x}) n_\infty^L(\mathbf{x}) - v_0(\mathbf{x}) n_0^L(\mathbf{x}) + \rho e^{-\rho t} v_t(\mathbf{x}) n_t^L(\mathbf{x}) - e^{-\rho t} n_t^L(\mathbf{x}) \dot{v}_t(\mathbf{x})\Big) d\mathbf{x}
$$

Similarly

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} y_t(\mathbf{x}) \dot{n}_t^F(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(y_\infty(\mathbf{x}) n_\infty^F(\mathbf{x}) - y_0(\mathbf{x}) n_0^F(\mathbf{x}) + \rho e^{-\rho t} y_t(\mathbf{x}) n_t^F(\mathbf{x}) - e^{-\rho t} n_t^F(\mathbf{x}) \dot{y}_t(\mathbf{x})\Big) d\mathbf{x}
$$

For the choices to be optimal, any perturbation to distribution $n_t^L(\mathbf{Q})$ must yields no change to the Lagrangian. This implies:

$$
\begin{aligned}
(\rho + g_t)\, v_t(\mathbf{x}) =& e^{z+\alpha+\beta} \zeta_t \underbrace{\frac{Q_t}{Q_t^F}}_{=1+\phi_t} + \iota(\bar{\beta} - \beta) \frac{\partial v_t}{\partial \beta}(\mathbf{x}) + \dot{v}_t(\mathbf{x}) && (46) \\
& + \max_{\theta,\tau} \lambda(\theta) \mathbb{E}_{\gamma'} \Big[u_t(\mathbf{x}') + y_t(\mathbf{x}') - v_t(\mathbf{x})\Big] - \theta \kappa_s \frac{\mathbf{w}_t}{\mathbf{C}_t} && (47) \\
& && (48)
\end{aligned}
$$

For the choices to be optimal, any perturbation to distribution $n_t^F(\mathbf{Q})$ must yields no change to the

Lagrangian. This implies:

$$(\rho + g_t)\, y_t(\mathbf{x}) = -\, e^{z+\beta}\zeta_t\phi_t(1+\phi_t) + \iota(\bar{\beta}-\beta)\frac{\partial y_t}{\partial \beta}(\mathbf{x}) + \dot{y}_t(\mathbf{x}) \tag{49}$$

The other choices follow its first order condition:

$[\eta_t]$

$$D'(\eta_t)\frac{\mathbf{w}_t}{\mathbf{C}_t} = \mathbb{E}_{\beta_0} v\Big((\beta_0, 0, 0)\Big) \tag{50}$$

$[\phi_t]$

$$\zeta_t = \Pi'(\phi_t) \tag{51}$$

Combining these equations we reach the results in main text.

C.3 Model Proofs and Discussion

Proof of Lemma 1. The equilibrium condition defines a two equation system in terms of (μ, s):

$$\mu - \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1} = 0$$

$$s - \frac{\phi\mu^{1-\sigma}}{\phi\mu^{1-\sigma}+\bar{\mu}^{1-\sigma}} = 0$$

Totally differentiating this system with respect to ϕ we reach the following linear system:

$$\begin{bmatrix} 1 & -\frac{1}{(1-s)^2(\sigma-1)} \\ (\sigma-1)s(1-s)\frac{1}{\mu} & 1 \end{bmatrix} \begin{bmatrix} \frac{d\mu}{d\phi} \\ \frac{ds}{d\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ -s(1-s)\frac{1}{\phi} \end{bmatrix} = 0$$

Inverting this linear system we get

$$\begin{bmatrix} \frac{d\mu}{d\phi} \\ \frac{ds}{d\phi} \end{bmatrix} = \frac{\sigma(1-s)+s}{\sigma} \begin{bmatrix} \frac{s}{(\sigma-1)\phi(1-s)} \\ s(1-s)\frac{1}{\phi} \end{bmatrix}$$

By definition $\Pi(\phi) = \left(1 - \frac{1}{\mu(\phi)}\right) s(\phi)$. Taking the logarithm:

$$\log \Pi(\phi) = -\log\left(\sigma(1-s(\phi)) + s(\phi)\right) + \log s(\phi)$$

The log-differential is

$$\frac{\Pi'(\phi)}{\Pi(\phi)} = \left[\frac{\sigma-1}{\sigma(1-s)+s} + \frac{1}{s}\right]\frac{ds}{d\phi} = (1-s)\frac{1}{\phi}$$

where the second equality uses the result from matrix inversion. This finishes the proof that:

$$\frac{\Pi'(\phi)\phi}{\Pi(\phi)} = 1 - s(\phi)$$

Proof of Proposition 3. To reach the aggregation result, we aim to write the real consumption as a function of production labor input $\mathbf{L}_P$, aggregate appeal $\mathbf{Q}$, and an aggregate efficiency of labor allocation $\mathbf{A}$. We start from the aggregation within product group k. Within product group k, the total group-level expenditure is $\alpha_k \mathbf{C}(t)$. Using the formula for sales shares, the expenditure for leader is

$$\frac{\phi\mu(\phi)^{1-\sigma_k}}{\phi\mu(\phi)^{1-\sigma_k} + \bar{\mu}^{1-\sigma_k}} \alpha_k \mathbf{C}(t) \tag{52}$$

Using the accounting equation for profit $\alpha_k \mathbf{C}(t) = \mu(\phi)\mathbf{w}(t)L_P$, we write that:

$$L_k(t) = \frac{\phi_k\mu(\phi_k)^{-\sigma_k}\frac{1}{Z_L(t)} + \bar{\mu}_k^{-\sigma_k}\frac{1}{Z_F(t)}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} \frac{\alpha_k \mathbf{C}(t)}{\mathbf{w}(t)}. \tag{53}$$

$$Z_k(t)^{\frac{1}{\sigma_k - 1}} L_k(t) = \frac{\phi_k\mu(\phi_k)^{-\sigma_k}\frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_L(t)^{\frac{1}{\sigma_k-1}}} + \bar{\mu}_k^{-\sigma_k}\frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_F(t)}^{\frac{1}{\sigma_k-1}}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} \frac{\alpha_k \mathbf{C}(t)}{\mathbf{w}(t)}. \tag{54}$$

Adding across all product groups

$$\mathbf{L}_P(t) = \frac{\mathbf{C}(t)}{\mathbf{w}(t)} \int_0^1 \alpha_k \frac{\phi_k\mu(\phi_k)^{-\sigma_k} + \bar{\mu}_k^{-\sigma_k}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} dk. \tag{55}$$

Search Process Discussion. In this section, we characterize the partial equilibrium in the search and matching markets, given (ϕ_k, Z_k) and the gains from reallocation across firms. Specifically, let $u_k(\beta, \gamma)$ be the discounted value of a fringe firm with product quality β and match quality γ, let $v_k(\beta, \gamma)$ be the discounted value of an additional product to the leader, and let $x_k(\beta, \gamma)$ be the discounted loss of an additional product operated by the leader in the calculation of leaders.

When positive buying flows into fringe firms occur, the optimal buying decision of a fringe firm with (β, γ) is as follows:

$$\kappa^s \varphi_0 = \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\Delta} \left[u(\beta, \gamma_L + \Delta) - \tau \right]^+, \tag{56}$$

s.t.

$$\lambda(\theta) \mathbb{E}_{\gamma_L} \left[u(\beta, \gamma_L) - \tau \right]^+ = U^F(\beta, \gamma).$$

It is straightforward to show Equation (56) is equivalent to the following problem in terms of solutions:

$$U^F(\beta,\gamma) = \max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'_L}\Big[u(\beta,\gamma+\Delta) - u(\beta,\gamma)\Big]^+ - \theta\kappa^s\varphi_0 \tag{57}$$

Equation (57) provides an intuitive interpretation of the reallocation process: due to directed search and the competition on the buyer side, the terms of trade aims to maximize the net benefit of reallocating products from fringe firms to other fringe firms, taking into consideration of the search friction and the cost of search. It is also worth noting that for each (β,γ), Equation (57) can be independently solved without referring to the distribution of products across firms. This mechanism is the block recursivity highlighted in Menzio and Shi (2011).

Similarly, due to free entry of fringe buyers, the leader-to-fringe (LtF) flows can be characterized in the same way. For notational simplicity, we define the joint surplus of reallocating a product from fringe to leader as $\Omega(\beta,\gamma_L,\gamma_F)$. The equilibrium in the LtF market is characterized by $\{U^L(\beta,\gamma),\theta^{LF}(\beta,\gamma)\}$ that jointly solve the following problem:

$$U^L(\beta,\gamma) = \max_{\theta} \lambda(\theta)\mathbb{E}_{\Delta}\Big[-\Omega(\beta,\gamma,\gamma+\Delta)\Big]^+ - \theta\kappa^s\varphi_0. \tag{58}$$

The reallocation flow from the fringe to leaders is more complicated because there is no longer free entry on both sides of the market. However, the leader as a buyer faces competitive pressure from fringe buyers. In an equilibrium where flows are observed, the leader must offer the same expected value of selling as the fringe buyers. Thus, the optimal buying decision of the leader is

$$\kappa^s\varphi_0 \leq \max_{\tau} \frac{\lambda(\theta)}{\theta}\mathbb{E}_{\gamma_L}\Omega(\beta,\gamma',\gamma_F) - \frac{1}{\theta}U^F(\beta,\gamma_F). \tag{59}$$

C.4 Derivation of Killer Acquisition Threshold

The killer acquisition threshold occurs when the value of selling a brand to the leader is negative, regardless of the efficiency differential between leader and fringe firm. This same intuition delivers a situation where leaders want to buy brands regardless of how efficient they would be at deploying the brand.

To theoretically study this situation, we focus on the relationship the leader and fringe have to a fringe firm's brand. We first take the difference between $u_t(\mathbf{x})$ (the value of brand to fringe firm) and $y_t(\mathbf{x})$ (the value of a *fringe's* brand to leader):

$$(\rho+g_t)\left[-y_t(\mathbf{x})-u_t(\mathbf{x})\right]=\underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\left[\Pi'(\phi)-\pi(\phi_t)\right]}_{\text{Operating Profit}}+\underbrace{\iota(\bar{\beta}-\beta)\left[-\frac{\partial y_t}{\partial\beta}(\mathbf{x})-\frac{\partial u_t}{\partial\beta}(\mathbf{x})\right]}_{\text{Maturity}} \tag{60}$$

$$+\underbrace{\max_{\theta}\lambda(\theta)\mathbb{E}_{\gamma'}\left[\Omega_t(\mathbf{x}',\mathbf{x})\right]^{+}-\theta\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}}+\left[-\dot{y}_t(\mathbf{x})-\dot{u}_t(\mathbf{x})\right]$$

First we find a threshold of ϕ such that $\Pi'(\phi)>\pi(\phi)\frac{1}{\phi}$:

$$(1-s(\phi))\frac{\Pi(\phi)}{\phi}>\frac{1-s(\phi)}{\sigma\phi}$$

$\iff$

$$\frac{s(\phi)}{\sigma(1-s(\phi))+s(\phi)}>\frac{1}{\sigma}$$

$\iff$

$$s(\phi)>\frac{\sigma}{2\sigma-1}$$

Whenever the market share of leader is above this threshold, it must be $-y_t(\mathbf{x})-u_t(\mathbf{x})>0$. Thus the gains from trade $\Omega(\mathbf{x}',\mathbf{x})$ is positive for any combination $(\mathbf{x}',\mathbf{x})$. As a result, there is never gains from trade of reallocating a product from leaders to fringes.

D Estimation Appendix

In this section, we discuss in greater detail the estimation process, starting generally and then discussing the different ingredients central to our estimation. 1. We directly calibrate the substitution elasticity σ_k to the ones estimated in the literature; 2. Given ϕ_k, we jointly estimate $\{\kappa_s,\kappa_e,d_k\}$ that minimize the distance between the observed reallocation rate, ϕ_k, and leader's innovation rate, as well as fringe firms' innovation rate.

D.1 Solving Equilibrium Given Parameters

Given any set of parameters, we take the following steps to solve the equilibrium, working at the group and aggregate level:

G1. *(Group Loop - Value Function)* For a fixed $\left(\frac{\mathbf{w}}{\mathbf{C}},\phi,g\right)$, we solve the balanced growth path value functions and decisions according to the Bellman equations discussed in the main text. This can be done using any PDE solvers. We used the finite difference method;

G2. *(Group Loop - Aggregation)* Given the decisions, we solve for the BGP distribution, scaling the fringes' entry rate such that the BGP quality gap is consistent with the imputed ϕ. With this distribution, we calculate the residual in the free-entry condition and the residual in growth decomposition.
G3. *(Group Loop - Equilibrium)* We repeat step 1 and 2 such that the residuals on free entry condition and growth decomposition are both close enough to zeros.
A1. We repeat G1 - G3 for all groups, given a guess $\frac{\mathbf{w}}{\mathbf{C}}$. Using the aggregation results, we solve for the aggregate search labor, innovation labor, and production labor.
A2. Repeat A1 until the guessed $\frac{\mathbf{w}}{\mathbf{C}}$ are close enough to the one implied by labor supply curve.

D.2 Estimating Parameters

There are four parameters to estimate for each product group: two innovation costs and two search costs. For each set of parameter values, we solve the equilibrium, and calculate the aggregate innovation rate and reallocation rate for leaders and fringes. We find the parameters by the method of moments by minimizing the absolute norm between the model predicted rates and rates from data.

Elasticities, Shares, and Markups. At the inner layer, we need to establish the value functions of each agent and do value function iteration to link the shares and elasticities with the optimization problem of the leader and the fringe entry and selling decisions.

We start by specifying the leader's perceived elasticity, as discussed in the model, and in Equation (61),

$$\epsilon(s) = (\sigma(1-s) + s). \tag{61}$$

This simultaneously delivers a markup, of a leader with share s and a standard markup $\bar{\mu}$ for the fringe firm in Equation (62),

$$\mu(s) = \frac{\epsilon(s)}{\epsilon(s)-1} \;\; ; \;\; \bar{\mu} = \frac{\sigma}{\sigma-1}. \tag{62}$$

We also can specify the share as a function of the leader quality advantage ϕ, as follows:

$$s(\phi) = \max(1 + \phi^{-1}(\sigma/(\sigma-1)/\mu(x))^{1-\sigma}), (0,1)) \tag{63}$$

$$\Pi_{fringe}(\phi) = 1/\sigma(1+\phi)(1-s(\phi)) \tag{64}$$

As a result, we can link the leader concentration ϕ to market shares and the elasticities firms face. This will represent the inner layer of our model, which occurs inside each iteration.

Full Discussion of Estimation. For more granular details of estimation, please see liangjiewu.com/files/tm_pw_apx_oct22.pdf

E Quantitative/Policy Discussion

In our quantitative exercises, we focus on different policies that seem to send a general message on brand reallocation. First, due to significant leader appeal and sales movement after exchange, we expect brand reallocation to show efficiency gains. We find this in the model, and find that downstream innovation also responds positively. Second, due to the age profile, the reallocation has less of an effect on growth than in markets with a faster age profile. Third, subsidizing entry is a more effective means of pursuing a reduction in concentration, as it simultaneously solves the growth and concentration externalities.

We believe that these results are robust to various specifications. First, on shutting down or taxing the reallocation of brands, we observe the responsiveness of brands to leader appeal is consistently larger than 0.4, while the marginal cost of leaders appear to be similar with fringe firms (Hottman et al., 2016). Leaders do engage in strategic behavior, but policy that shuts down reallocation will lose out on these gains and the forward looking behavior of fringe firms. This is attenuated if the leader appeal advantage declines or the brand maturity slows.

Second, as we see in Table 11, varying the maturity of brands has significant effects on policies, as faster maturity links innovation and reallocation more tightly together. This comes from directed search, and is consistent regardless of the life cycle characteristics, as long as there is some time to maturity, which is consistent with our paper and other work in the literature (e.g., Bronnenberg et al., 2009).

Third, subsidizing entry is a good policy for both attenuating concentration and increasing growth. This should hold as long as fringe firms have an innovation advantage (relative to their size) to leaders. If policy subsidizes product entry, both fringe and leading firms response, but fringe firms are able to respond more strongly. Even with reallocation, the steady-state share of fringe firms holdings are higher because reallocation occurs later in life. As a result, we feel the main messages of policy are robust to different specifications, but we look forward to further empirical and quantitative work to further explore these mechanisms.