Matching in Teams and Wages: A Quantitative Framework

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Abstract

This paper explores sorting and the wage structure through the lens of teams. A firm collects workers of differing roles to produce in teams, subject to a noisy team assembly cost. The equilibrium captures the intuition behind the classic Becker (1973) matching model, yet this model requires fewer restrictions on the production function. Further, it allows for a tractable framework for analyzing large teams in firms. The model admits applications, such as wage inequality and tax policy. We discuss two applications through the lens of this framework: the effect of production complementarities on the distribution of wages and the spillover effect of taxation on co-workers. The production externality in teams and team assembly cost provide both a new framework for standard applications and tractability.

1 Introduction

Modern production processes are organized in teams. Workers differ from each other both horizontally (their role/occupation in teams) and vertically (their skill within this role). The distribution of these skills and the complementarity or substitutability among these roles are important for understanding the wage structure of the economy. Matching models may be amenable to addressing an exploration of these complementarities. However, matching models since Becker (1973) direct attention mostly to matching between firms and workers. These models assume away complementarities between workers in different roles.

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This paper builds a framework of teams in firms that is amenable to salient features of modern teams. First, there are many potential roles on a team. Second, there is vertical differentiation within these roles. Third, the cross-role complementarities are heterogeneous across types. For instance, managers may add more value to skilled workers than unskilled workers through better understanding subtle strategic and time management practices. Thus, this project has immediate data applications.[1]

This paper builds a model that rationalizes heterogeneous wage correlations among occupation pairs in a matching framework. In the model, firms hire workers of different occupations from competitive labor markets to form teams. Each team is composed of one worker from every occupation. Workers differ in their abilities within their occupation. Whether high-wage workers tend to work together depends not only on the pairwise complementarities as in Becker (1973), but also the entire complementarity structure of team production function and the skill distributions within each type.

Matching more than two occupations is not a trivial task. Classical conditions from matching theory (e.g., modularity of production) that guarantee existence of equilibrium do not apply in the general setting.[2] We introduce a team assembly cost to gain tractability. The team assembly cost is proportional to the concentration of the distribution of types within a firm. In order to assemble a more complementary team, firms must pay a higher cost to target their assembly process. This cost captures the time and effort in the recruiting process to figure out the abilities of workers and how they complement each other. The firm's decision problem balances the motive of revenue maximizing (better teams) and the motive of cost saving (harder to assemble the right team). The resulting input decision is partially targeted: Firms form teams that generate higher output with higher probability due to the revenue motive, but there remains a degree of randomness due to the cost motive.

The partially targeted input decision generates non-pure matching among occupations. High ability workers sometimes work with low-ability coworkers because it is costly for their employers to match teams perfectly. This result captures the fact that the rank correlation across occupations within firms is in between 0 and 1. Second, it simplifies the characterization of an equilibrium of many-agent matching. The cost of forming a team turns an extensive input decision (which team composition) into an intensive-margin decision (how many of each team composition). We establish the existence of a matching equilibrium with multiple occupations. An integral equation system charac-

[1]We have a current dataset under disclosure review to illustrate the benefits of this framework and confirm the priors of the model.

[2]Mathematically, it extends the two-dimensional Monge-Kantorovich problem into multi-marginal problem.

terizes the allocations and the wages from this matching equilibrium. This equilibrium system is tractable and fast algorithms have been developed to solve it (e.g., Knight, 2008; Cuturi, 2013).

We further parametrize the model in order to provide a framework for data applications. We assume (1) there is a common ranking of worker abilities (high type workers are more productive in every team) and (2) abilities of workers enter production as a quadratic function. We derive closed-form solution to the slope of wage profile within each occupation with these two assumptions. The wage difference between high-type and low-type workers within each occupation depends linearly on their own ability and the expected rank of their coworkers of different roles. The coefficient in front of each coworker occupation measures the complementarity between their abilities. This closed-form relationship allows for a flexible characterization of complementarity among occupations.

Using matched employer-employee data, a researcher can take individuals within an occupation and observe the wage distribution at the firm across occupation and skill rank. The rank correlation across all firms between the rank of each occupation delivers a natural result on sorting at firms across individuals. In particular, this empirical approach allows a researcher to characterize heterogeneous sorting across types. Characterizing the rank correlation at firms requires no economic theory. However, economic theory is key understanding how these heterogenous sorting patterns across types transmit to wages.

Workers that provide significant complementarities to other workers will receive a higher wage as the firm assembles the team. The theoretical result allows for simple functional forms that can be taken to the data. When evaluating data counterparts, the estimation of the wage function does not require information on the cost of forming teams. The cost of forming teams only matters for counterfactual analysis, but not for evaluating how the complementarity structure of team production transmits to wages.

In our specified production function and estimated responsiveness, we then explore policy applications. We start with the introduction of a progressive tax on managers. The ripple effects of the policy depend on the strength of sorting and estimated responsiveness to coworkers.

This paper makes three main contributions. First, we provide a framework to study team formation among workers with heterogeneous skills. The equilibrium framework is tractable enough to accommodate rich assumptions on the distribution of abilities and assumptions on the complementarity and substitutability among occupations. Second, we document how this framework can easily be brought to data to evaluate a host of issues. Third, this paper highlights the spillover effects through complementarities in production. Technology shocks that improve certain occupation's productivity, or public policy that creates wedge for certain occupations, will transmit to other occupations through their complementarities and the matching pattern.

Related Literature

This paper is related to matching theory and the study of teams.

Economists have long been interested in matching and sorting both where individuals match with other individuals and where individuals match with firms. Gale and Shapley (1962) and Becker (1973) pioneered a large theoretical literature on matching without frictions. Becker (1973) provided the fundamental intuition behind the complementarities and sorting: without frictions, complementary types will match. Since Becker (1973), this work has been extended to a wide array of contexts.

While these models illustrated the importance of the underlying complementarities to the matching process, they didn't address frictions in the matching process. It is hard for people to perfectly target their optimal match in many markets. Since Stigler (1961), economists have tried to reconcile theoretically the idea of "imperfect sorting" through the lens of search frictions in individuals finding firms and matching with them. Chade et al. (2017) review this literature. and extend it to markets with search frictions.

We build on this literature in two ways. First, we endogenize the search and matching process simultaneously. The model builds rational inattention in the search and team assembly process as Wu (2019) did for the firm-worker matching process. Second, we move the theory away from worker-firm matches to worker and coworker matches. This provides a more intuitive link on sorting between types. Firms in this context simply provide the technology to match coworkers. Thus, the firm still matters in that they absorb search costs and facilitate teams but we do not need to attribute to them any fundamental value that is disconnected from the collection of agents that work for the firm.

Many papers deal with the problem of assigning rank or value to firms in the empirical and quantitative literature on sorting (Abowd et al., 1999; Bagger and Lentz, 2018). Because there is no obvious way of ranking firms, economists need to make certain assumptions that build a quality measure of a firm. This has been explored in a sorting framework in a host of papers (e.g. Abowd et al. (1999), Hagedorn et al. (2017) Lentz et al. (2018) Bagger and Lentz (2018)). These papers find that firms are indeed an important determinant of worker's wages. New papers, however, have questioned whether this is in fact the case (e.g. Bonhomme et al., 2019), which suggests direction on the effect of co-workers rather than firms is more important.

Another reason to direct attention to co-workers is the difficulty of ranking firms, which has received recent attention (Sorkin, 2018). When firms are ranked according to the wage premium they provide, it is hard to understand what this truly reflects about the "firm" component. As Lopes de Melo (2018) notes, the dependency on coworkers is likely a larger effect than firms. However, Lopes de Melo (2018) does not provide a structural framework to fulfill this task. In making our theoretical model tractable, we can overcome the abstract concept of the firm and more intuitively speak to the matching between coworkers.

Our theoretical framework builds on applications of transportation theory in economics. Ever since Monge (1781), economists and social scientists have thought through the problems of moving objects from one space to another. This has broad applications, and gained renewed interest with Kantorovich (1942) and more recently with Villani (2009). Economists have also noted the wide array of applications of transportation theory (Galichon, 2016). In the labor market, Lindenlaub (2017) uses applications from this framework to speak to worker-firm matches.

We make use of two unique tools from optimal transport theory. First, we apply multi-marginal optimal transport (Pass, 2014; Carlier et al., 2014). The key principle is that multi-marginal matching contains N (e.g. team) agents and maps them to a team rather than 2 (e.g. worker-firm). Many problems in economics are about many agents (i.e. > 3) matching, and thus our goal is to extend the insights from optimal transport theory to these environments.

The balancing of targeting search and reducing search cost enables a smoothing of the maximization problem. The basic principle contains the classic tradeoff in economics: balancing the returns of the optimum with the cost of targeting. In our scenario, there are optimal teams that entrepreneurs aim to assemble, but it is costly to assemble them with exact precision. Because of this cost, entrepreneurs will tend to sort people into the right teams but their ability to sort will be limited. This makes a model that would be complicated a potentially have many solutions unique and tractable.

2 Model

The goal of the model is to build a framework of team assembly with heterogeneous workers. The model is a generalization of Becker (1973), where heterogeneous workers are matched with heterogeneous firms. The environment has two important new features: First, we consider the matching of workers from multiple occupations (≥ 2);

Second, we introduce a cost of assembling teams that firms face in terms of matching optimally across occupations. Due to the cost, team assembly is noisy in the sense it deviates from the optimal allocation (net of cost). The noise in assembling teams allows us to characterize equilibrium of the model for generic production functions, and offer a parsimonious way of rationalize the mismatch pattern in data.

We first introduce the environment given some general production function for teams. We then discuss equilibrium. This general model serves as the foundation then for an exploration of specific functional forms on production.

2.1 Environment

There are measure 1 of firms in the economy. Each firm has measure 1 of teams to assemble and each team requires labor input from N occupations to produce output. The outputs are sold in a competitive market with price 1. For now, in order to isolate the matching process of occupations, we do not model the output market and the entry decision of firms.

For each of the occupations $n \in \{1, ..., N\}$, there are measure 1 of workers. They differ by their type $x_n \in [0, 1]$. We assume x_n is uniformly distributed, so x_n can be interpreted as quantile within occupation n's type distribution. A firm then produces output from a measure 1 of teams, where each team individually inputs $\mathbf{x}$ which is assembled across the N occupations:

$$f(\mathbf{x}) = f(x_1, ..., x_N)$$

.

It is helpful to amend this framework with a simple assumption. When the partial derivative with respect to occupation n is positive, $f_n(\mathbf{x}) \geq 0$, we assume workers are vertically differentiated. The team is more productive when it has a higher type x_n worker. In this case, x_n can be interpreted as the rank of productivity within occupation n. The output of different teams is perfectly substitutable within a firm. So, denoting the probability density function of the worker mix $\mathbf{x}$ within the given firm as $a(\mathbf{x})$, the total production of the firm is:

$$\int_{\mathbf{x}} f(\mathbf{x}) a(\mathbf{x}) d\mathbf{x}.$$

Two assumptions are crucial here: a firm only has measure 1 of teams to assemble and output is linear. Both assumptions are stark stands about reality, yet it follows the tradition of matching models and greatly simplifies our analysis. We also assume all people participate in the labor market (outside option is $-\infty$) in order to limit our focus to allocations across teams.

Workers are hired from N competitive labor markets, one for each occupation. Firms take as given $w_n(x)$, the wage for x type worker within occupation n. If the distribution of labor mix within the firm is $a(\mathbf{x})$, the total labor cost of the firm is $\sum_{n=1}^{N} \int_{\mathbf{x}} w_n(x_n) a_n(x_n) dx_n$, where $a_n(x)$ is the marginal distribution of $a(\mathbf{x})$ in dimension n.

The environment described so far is very similar to transferable utility matching models, except we are consider matching of $N \geq 2$ occupations . When there are only 2 occupations, it is identical to Becker's matching model. Now we introduce the key ingredient to make our model different. Our new addition here in order to get a tractable idea of team structure is a cost of assembling teams, which smooths out the problem and can be interpreted as an information processing cost. This entropy cost generates a degree of mismatch in the economy and extends the Becker model to deliver wages and allocations that depend on the processing cost as well as the set of potential matches in the economy.

Assume within occupation n's labor market, the types are distributed according to the uniform distribution.If a firm chooses to form teams according to the $a(\mathbf{x})$ distribution, it needs to pay a cost. The cost here is scalar c multiplied to Kullbak-Leibler divergence between the chosen team mix $a(\mathbf{x})$ and the exogenous distribution of worker types:

$$\text{cost of team assembling} = c \int_{\mathbf{x}} a(\mathbf{x}) \log a(\mathbf{x}) d\mathbf{x}.$$

The Kullbak-Leibler divergence is convex in $a(\mathbf{x})$ and is minimized when the firm chose the exogenous distribution. This cost introduces a motive for firms to stay close to type distribution of the market. The scalar c weighs cost saving motive against revenue maximization motive. When c is large, firms will stay closer to the market distribution. When c is small, firms stay closer to the revenue maximization decision, which is the case of matching models in the Beckerian spirit.

The entropy cost has both economic and mathematical roots. One economic story is rational inattention–it is highly costly for individuals to optimally allocate attention to building the perfect team. Other stories on a related note might be a path dependency in team structure or incomplete information on worker types. The reason we take a stand on the structure of the cost comes from standard practice in information theory. Individuals with limited information will tend to pull from the existing distribution in the market for agents. The mathematical origins are from information theory and have a broad array of applications in fields such as machine learning and image processing.

2.2 Planner's Problem

The social planner's problem illustrate the tradeoff in the environment. Consider a social planner that aim to maximize the total output net of the cost of team assembling:

$$\max_{a(\mathbf{x})} \int_{[0,1]^N} f(\mathbf{x})a(\mathbf{x})d\mathbf{x} - c\int_{[0,1]^N} a(\mathbf{x})\log a(\mathbf{x})d\mathbf{x},$$

s.t.

$$\int_{[0,1]^{N-1}} a(\mathbf{x})d\mathbf{x}_{-i} = 1, \quad \forall i.$$

The social planner's problem is a concave problem. It admits a unique interior solution that satisfies the following first-order condition in $(a(\mathbf{x}), \{V_i\}_i)$:

$$f(\mathbf{x}) - c\log a(\mathbf{x}) = \sum_{i=1}^{N} V_i^*,$$

$$\int_{[0,1]^{N-1}} a(\mathbf{x})d\mathbf{x}_{-i} = 1, \quad \forall i.$$

The socially efficient allocation equalized the marginal benefit of forming team, $f(\mathbf{x})$, to the marginal cost, $c\log a(\mathbf{x})$ to the sum of shadow value of worker supply $\sum_i V_i^*$.

2.3 Firm's Optimal Decision

We can now write out the firm's maximization problem. Each firm takes the wages of type x within occupation n as given $(w_n(x))$ and solves the problem of maximizing profits subject to paying out wages and a team assembly cost.

$$\max_{A(\mathbf{x})} \underbrace{\int_{[0,1]^N} f(\mathbf{x})a(\mathbf{x})d\mathbf{x}}_{\text{Production}} - \underbrace{\sum_{n=1}^{N}\int_0^1 w_n(x_n)da_n(x_n)dx_n}_{\text{Wage Bill}} - \underbrace{c\int_{\mathbf{x}} a(\mathbf{x})\log a(\mathbf{x})d\mathbf{x}}_{\text{Entropy cost}}.$$

A firm's optimization problem is strictly convex when $c > 0$, as the production and wage part is linear and the cost of assembling team is strictly convex in $A(\mathbf{x})$. The first order condition of firm's problem then is as follows:

$$a(\mathbf{x}) = \exp\left(\frac{f(\mathbf{x}) - \sum_n w_n(x_n)}{c} - 1\right).$$

The optimal input decision looks similar to an optimal input problem in standard models, adjusted by the entropy cost. The equation expresses how firm's labor demand function responds to the mix of workers. A firm will want to hire more labor than supplied from the market if the profit from this kind of mix, $a(\mathbf{x})$ is high. The cost of assembling teams governs how input decisions respond to profit. When c increases, the input decision is more detached from profit and firms assemble teams more randomly. We now turn our attention to the general equilibrium results that emerge from the entire measure of firms optimizing.

2.4 Equilibrium

All firms operate through assembling teams in a competitive labor market. In each market for occupation n, we look for a wage scheme $w_n(x)$ such that demand and supply for workers equalize. In our notation, it means the measure $A(\mathbf{x})$ and $V(\mathbf{x})$ are equal for any $\mathbf{x}$ with positive supply. Recall $A(\mathbf{x})$ and $V(\mathbf{x})$ are the distributions of worker supply mix within the firm and within the general market respectively.

The total demand for occupation n worker with type x is:

$$\int_{\mathbf{x}_{-n}} a(\mathbf{x}) d\mathbf{x}_{-n} = 1.$$

As all worker of type x_n in occupation n will participate with probability 1, if $w_n(x_n) > 0$. Almost surely, the left and right hand sides of the market clearing condition has to hold in terms of density for $\mathbf{x}$ with $w_n(x_n) > 0$. The market clearing condition in density requires:

$$\exp\left(-\frac{w_n(x)}{c}\right) \int_{\mathbf{x}_{-n}} \exp\left(\frac{f(x, \mathbf{x}_{-n}) - \sum_{n' \neq n} w_n'(x)}{c} - 1\right) d\mathbf{x}_{-n} = 1.$$

The following definition formally characterize the competitive equilibrium we are looking for.

Definition 1 (Competitive Equilibrium)
A competitive equilibrium is a tuple of $w_n(x)$ such that the following condition holds whenever $w_n(x) \geq 0$

$$\exp\left(-\frac{w_n(x)}{c}\right) \int_{\mathbf{x}_{-n}} \exp\left(\frac{f(x, \mathbf{x}_{-n}) - \sum_{n' \neq n} w_n'(x)}{c} - 1\right) d\mathbf{x}_{-n} = 1$$

Proposition 1 (Existence of equilibrium)
The competitive equilibrium exists.

Proof 1
Relabel $w_n(\mathbf{x})$ as the market price for "commodity" $\mathbf{x}$ and $f(\mathbf{x}) - c \log a(\mathbf{x})$ as the utility function for the firms. We can interpret the market equilibrium as an exchange economy for commodity $\mathbf{x}$. The existence of equilibrium can be established using results in classical work such as Ostroy (1984)

2.5 Wages

Our ultimate goal in this paper is to study the implications of this multi-occupational model on the wage structure of the economy. In this section, we show the model delivers a tractable wage structure, closely linked to information we observe from a matched employer-employee dataset. Let's focus on occupation n and an interval of worker type $(x, x + \epsilon]$ such that wage is positive. The market clearing condition needs to hold on density at these points. We can invert the market clearing condition to get the following equation for the wage:

$$w_n(x) = c \log \int_{[0,1]^{N-1}} \exp\left(\frac{f(x, \mathbf{x}_{-n}) - \sum_{-n} w_{-n'}(x)}{c} - 1 \right) d\mathbf{x}_{-n}. \tag{1}$$

There is a natural interpretation to Equation 1. The wage of a type x_n worker is equal to her productivity in an expected team. For instance, if x_n would tend to work with people of high and complementary ability, they will get paid a higher wage. The expectation is taken on the optimal set of teams that involve x workers. The Log-Sum-Exp function reflects the tendency for the firms to match workers with teams of high returns. Thus, an individual is paid in accordance with the expectation of the teams they would work with.

This gets at two points of intuition. First off, the team structure matters for wages because of potential complementarities. Second, an individual can get a higher wage if they have a tendency to team with complementary people even if they do not currently work with them. This is because individuals who can work with highly complementary individuals can thus command a higher wage, if indeed there are other entrepreneurs who want to poach them.

When $c \to 0$, all workers will be matched with their best team taken as given the wage function, we return to a generalized case of Beckerian matching models. When

$c \to \infty$, all workers are randomly matched according to their marginal distribution upon entry. Wage in this case is simply the average productivity over the random teams.

If we assume f is differentiable on the interval $(x, x + \epsilon]$, we can characterize the slope of this wage function then as follows:

$$w'_n(x) = \int_{[0,1]^{N-1}} a(x, \mathbf{x}_{-n}) f_n(x, \mathbf{x}_{-n}) \Pi_{-n} d\mathbf{x}_{-n}. \tag{2}$$

We see here how within occupation wage dispersion is driven by both the marginal productivity of workers, $f_n(x, \mathbf{x}_{-n})$. In addition, changes in an individual's ability changes the distribution of workers she matches with, $a(x, \mathbf{x}_{-n})$. This is the team effect. As individuals abilities improve, firms want to put them in more appropriate teams. This will then come back through the wage.

We have shown from a generalized framework the driving forces behind wage dispersion within types depending on their own productivity and the teams they join. To more concretely take this production process to data we will employ some specific production functions that allow us to tease out the coworker and own-worker effect on wages.

3 Specific Production Functions

In this section, we take two specific production functions to extend the model to bring it to the data. In exploring these two cases, one case will have production exhibit no complementarities across types and a second example where we embed the complementarities across types. Our goal is to present these two models to understand what we are missing when we evaluate how wages operate only at the individual type level. The parametrization with the quadratic production function in Section 3.2 also allows us to explore data counterparts in closed form.

3.1 Example 1: Production without complementarities

Assume production is Cobb-Douglas in all roles:

$$f(\mathbf{x}) = \sum_n \alpha_n \log x_n.$$

The wage function becomes:

$$w_n(x) = \alpha_n \log x_n + c \log \int_{[0,1]^{N-1}} \exp\left(\frac{\sum_{-n} \alpha_{-n} \log x_{-n} - \sum_{-n} w_{-n'}(x)}{c}\right) d\mathbf{x}_{-n}.$$

The level of wage is indeterminate as in the Becker case, but we can look for a "equal treatment" allocation WLOG

$$w_n(x) - \alpha_n \log x_n = 0.$$

In this case all workers are paid by their marginal product and their is no spillover of skills from coworkers.

3.2 Example 2: Quadratic Production

In this example, we assume a simple quadratic production function where complementarities are pairwise by type.

$$f(\mathbf{x}) = \sum_{n=1}^{N} \sum_{m=1}^{n} \sigma_{nm} x_n x_m.$$

Guess the entry rule on the worker's end is a threshold rule by $\underline{x}_n$. The slope of wage function becomes

$$w_n'(x) = \int_{\Pi_{-n}[\underline{x}_j, 1]} a(x, x_{-n}) (\sum_{m \neq n}^{N} \sigma_{nm} x_m) d\mathbf{x}_{-n}.$$

Rearranging terms we reach

$$w_n'(x) = \underbrace{2\sigma_{nn} x}_{\text{own effect}} + \underbrace{\sum_{m \neq n}^{N} \sigma_{nm} \int a_{nm}(x, x') x' dx'}_{\text{team complementarity effect}}. \quad (3)$$

Where $\int a_{nm}(x, x') x' dx'$ is the expected type of m-occupation that's matched with type x in n-occupation. This parametric case is often adopted by quantitative models as quadratic functions in generally match data well. Equation 3 has clear economic intuition: the premium earned by a worker is attributed to his own type, and the teammates he is expected to match with in equilibrium. The weights are determined by complementarity coefficients in production. This specific case allows for a tractable exploration of these coefficients, which we turn to now.

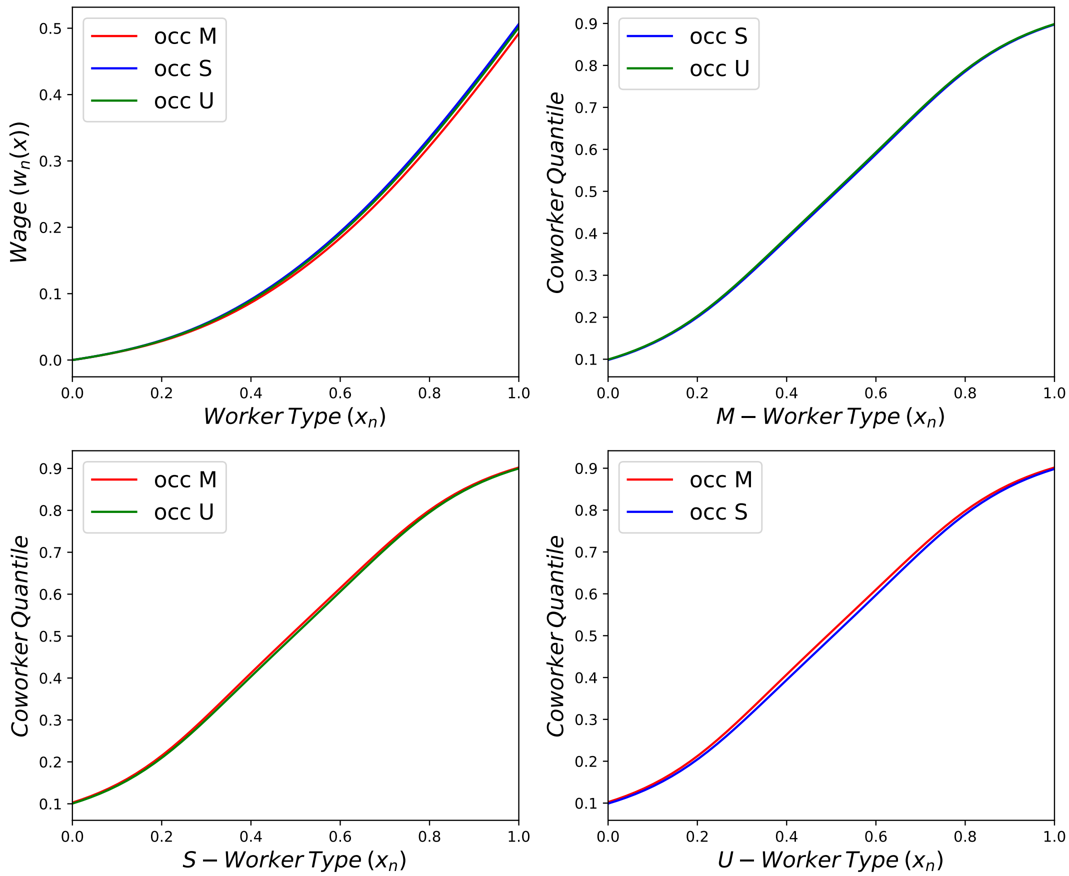
With the quadratic production

$$w_n'(x) = 2\sigma_{nn} x + \sum_{mn} \sigma_{nm} \bar{x}_m$$

The slope of wage function at quantile x of occupation n is a function of two forces. First, the individual's own marginal productivity (production weight $\times$ skill) will determine the wage. Second, the complementarity with coworkers (complementarity $\times$ expected skill of coworkers) will induce firms to sort specific people to complementary co-workers. The degree of this sorting strength will determine the team effect on wages. This has immediate potential data applications.

With MEE datasets we observe $w_n(x)$ and $\bar{x}_m$, can estimate σ_{nm}

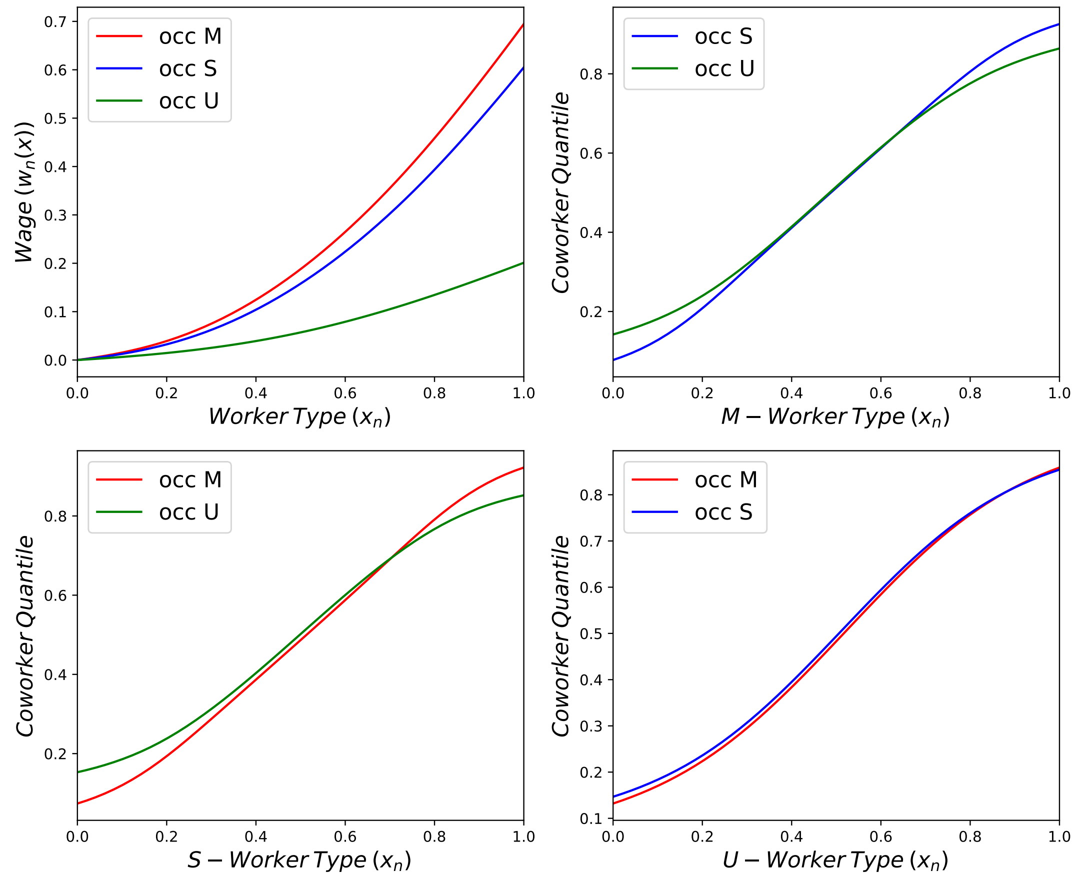
To explore the mechanism at work here, we plot out the wage function and matching pattern of a case of 3 occupations. Call the 3 occupations manager (M), skilled workers (S), and unskilled workers (U). Figure 1 plot out the case all occupations enter production in the symmetric way. The solid line is a case when cost of assembling team is low, while dotted line is a case when cost is high. The first subfigure shows the wage profile of workers along different skill level. The shape is convex, reflecting both the change in self-productivity and complementarity to other occupations. Subfigure 2-4 shows the expected quantile of coworkers. As occupations are symmetric, we will focus on figure 2. We can see more skilled managers are matched with more skilled skilled and unskilled workers, similar to the Beckerian models. Yet due to the cost of assembling teams, the matching is not perfect. Compare the case of low cost to high cost (solid to dash lines). When cost of assembling team increases, matching becomes noisier and wage is less convex in skills.



Production function $f(m, s, u) = ms + mu + su$

Figure 1: Symmetric Production with 3 Occupations

To understand how the complementarity in production affects sorting strength, we consider the case where workers enter production in a symmetric way. In this case, we assume managers and skilled workers are more complementary. Compare Figure 2 to Figure 1, we observe managers and skilled workers have steeper wage profile. The sorting strength is stronger between managers and skilled workers, compared to sorting between managers and unskilled workers. Notice the differential in sorting strength is more salient when cost of assembling team is higher.



Production function $f(m, s, u) = 2ms + mu + 0.5su$

Figure 2: Asymmetric Production with 3 Occupations

4 Quantitative Analysis with MEE Data

[Under Data Disclosure Check]

5 Policy Application: Progressive Taxation

This model draws a host of potential policy applications. We explore here an increase in progressive taxation. How do increases in taxes on high income workers affect their coworkers? In this framework, taxes will have direct effects on individual workers, but also indirect effects through the team formation process.

Overall, the incidence of taxation will depend on the degree of complementarities across co-workers and the cost of team assembly. Suppose the labor cost for occupation m is taxed by τ (e.g. wedge).

$$a(\mathbf{x}) = \exp\left(\frac{f(\mathbf{x}) - \sum_{n\neq m} w_n(x_n) - (1+\tau)w_m(x_m)}{c}\right). \tag{4}$$

Note immediately the tax on the wage transmits to the team structure $a(\mathbf{x})$. This will induce general equilibrium effects on wages across the distribution, as wages are linked to the assignment. We further explore this with the labor market clearing condition for occupation n:

$$\exp\left(\frac{w_n(x)}{c}\right) = \int_{[0,1]^{N-1}} \exp\left(\frac{f(\mathbf{x}) - \sum_{n\neq m,n} w_n(x_n) - (1+\tau)w_m(x_m)}{c}\right) d\mathbf{x}_{-n}.$$

We now take the first order condition with respect to a change in the tax rate to occupation m on occupation n with the rate fixed at $\tau = 0$:

$$\begin{aligned}\frac{\partial w_n(x)}{\partial \tau}|_{\tau=0} = &\underbrace{-\int_0^1 a_{nm}(x,x')w_m(x')dx'}_{\text{Direct Effect (-)}} \\ &\underbrace{-\sum_{n'\neq n}\int_0^1 a_{nn'}(x,x')\frac{\partial w_{n'}(x')}{\partial \tau}dx'}_{\text{Indirect Effect (-/+)}}.\end{aligned}$$

The above equation expresses the connection between taxes to a type m and the wage function for another type n. First, we dub the direct effect as observing how the tax changes the distribution of workers across firms $a_{nm}(x,x')w_m(x')$. If worker n is with less productive, or well-matched, type m's this will lower the wages. Second, there is the indirect effect in the sense that the tax on m changes the wage of all other types in firms. Let us discuss an example to make this more clear.

Suppose a tax is increased on managers in a world where firms have managers, low-skilled workers, and high-skilled workers. This will interact with the firm assembly through Equation 4. As such, the loss of efficiency in the allocation will induce firms to pay less for workers across the board. However, there is an indirect effect in the sense that low-skill workers may be matched to better high-skilled workers. If so, the overall effect on wages could be positive. If we apply the specific data-applicable functional forms, this enables simple regressions on tax events to shed light on how spillovers in the labor market transmit.[3]

6 Conclusion

This paper develops a new theory of teams and applies it to matching across coworkers in firms. We build on the intuition of Becker (1973) but bring in applications from information theory in order to smooth out the problem of many-person teams. We utilize entropy-regulated optimal transport which turns an extensive margin problem (who goes where) into an intensive margin problem – which depends on how easy it is to assemble teams.

The model lends itself to quantitative evaluation of the degree of the importance of coworkers on an individual's wage. Once we have these forces in hand, we can do policy counterfactuals with changes in tax rates on certain workers. We plan to explore these forces further in a later edition of this paper and report the connection between the theoretical framework and the data discoveries. This data is under disclosure review.

[3] We explore this in a data counterpart to this paper which is currently under disclosure review

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A Appendix

A.1 Example 2: 2-D with Normal Quantile

Assume production a product of normal quantile

$$f(x,y) = \sigma\Phi^{-1}(x)\Phi^{-1}(y)$$

Let's first solve for the allocation

$$a(x,y) = \exp\left(\frac{\sigma\Phi^{-1}(x)\Phi^{-1}(y) - w_1(x) - w_2(y)}{c}\right)$$

Guess the wage function takes the following form

$$w_n(x) = \alpha\Phi^{-1}(x)^2 + \frac{\beta}{2}$$

Our goal now is to find α and β such that the resource constraints hold. We then plug the conjecture back to assignment we have

$$a(x,y) = \exp\left(\frac{\sigma\Phi^{-1}(x)\Phi^{-1}(y) - \alpha\Phi^{-1}(x)^2 - \alpha\Phi^{-1}(y)^2 - \beta}{c}\right)$$

As the problem is symmetric we just need to focus on the first dimension

$$1 = \int_0^1 a(x,y)dy$$

We will make the change of variable $\tilde{y} = \Phi^{-1}(y)$. The constraint is now written as

$$1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(\frac{\sigma\Phi^{-1}(x)\tilde{y} - \alpha\Phi^{-1}(x)^2 - \alpha\tilde{y}^2 - \frac{c}{2}\tilde{y}^2 - \beta}{c} \right) dy$$

Our next step is complete the square

$$1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(\frac{-(\sqrt{\alpha + \frac{c}{2}}\tilde{y} - \frac{\sigma}{2\sqrt{\alpha+\frac{c}{2}}}\Phi^{-1}(x))^2 + (\frac{\sigma^2}{4(\alpha+\frac{c}{2})} - \alpha)\Phi^{-1}(x)^2 - \beta}{c} \right) dy$$

Notice the terms in bracket is proportional to a normal density with standard deviation

$$2\sqrt{\frac{c}{\alpha + \frac{c}{2}}}$$

So the integral is

$$1 = 2\sqrt{\frac{c}{\alpha + \frac{c}{2}}} \exp\left(\frac{(\frac{\sigma^2}{4(\alpha+\frac{c}{2})} - \alpha)\Phi^{-1}(x)^2 - \beta}{c} \right)$$

Now we need to find α and β solving

$$0 = \frac{\sigma^2}{4(\alpha + \frac{c}{2})} - \alpha$$

$$\frac{\beta}{c} = -\log 2\sqrt{\frac{c}{\alpha + \frac{c}{2}}}$$

We get

$$\alpha = \frac{-c + \sqrt{c^2 + 4\sigma^2}}{4}$$

A.2 Gaussian-Quadratic Case: Generalization to N-Dimensions

We extend the 2-dimensional case to N-dimensions. Suppose the production function takes the following form

$$f(\mathbf{x}) = 2\sum_{i \neq j} \sigma_{ij}\Phi^{-1}(x_i)\Phi^{-1}(x_j)$$

We guess the following quadratic function of wages

$$w_i(z) = \alpha_i\Phi^{-1}(z)^2 + \beta/N$$

According to the allocation function

$$a(\mathbf{x}) = \exp(\frac{2\sum_{i\neq j}\sigma_{ij}\Phi^{-1}(x_i)\Phi^{-1}(x_j) - \sum_i \alpha_i \Phi^{-1}(x_i)^2 - \beta}{c})$$

With the transformation

$$\tilde{x} = \Phi^{-1}(x)$$

$$a(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{N}{2}}}\exp(\frac{2\sum_{i\neq j}\sigma_{ij}x_i x_j - \sum_i(\alpha_i + \frac{1}{2})x_i^2 - \beta}{c})$$

Lets write everything in matrix form

$$\sum_{i\neq j}\sigma_{ij}x_i x_j = \mathbf{x}^T\Sigma\mathbf{x}$$

$$\sum_i(\alpha_i + \frac{1}{2})x_i^2 = \mathbf{x}^T\Omega\mathbf{x}$$

$$\Sigma = \begin{pmatrix} 0 & \sigma_{12} & ... & \sigma_{1N} \\ \sigma_{12} & 0 & ... & \sigma_{2N} \\ ... & ... & ... & ... \\ \sigma_{1N} & \sigma_{2N} & ... & 0 \end{pmatrix}$$

$$\Omega = \begin{pmatrix} -(\alpha_1 + \frac{1}{2}) & 0 & ... & 0 \\ 0 & -(\alpha_2 + \frac{1}{2}) & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & -(\alpha_N + \frac{1}{2}) \end{pmatrix}$$

The goal is to find vector α and β so that the market clearing condition holds dimension-wise.
$\forall i$

$$1 = \int_{-i} a(\mathbf{x})d\mathbf{x}$$

Fix dimension i, we can write the kernel as

$$\mathbf{x}^T(\Sigma + \Omega)\mathbf{x}. = \mathbf{x}_{-i}^T(\Sigma_{N-1,N-1} + \Omega_{N-1,N-1})\mathbf{x}_{-i} + 2x_i\sum_j \sigma_{ij}x_j - (\alpha_i + \frac{1}{2})x_i^2 \tag{5}$$

where the $N-1, N-1$ notation stands for deleting i-th row and i-th column of a matrix. We want to write the quadratic form in the following form

$$(\mathbf{x}_{-i} - x_i\mathbf{k}_1)^T M(\mathbf{x}_{-i} - x_i\mathbf{k}_1) + k_2 x_i^2$$

Expanding the quadratic form we have

$$\mathbf{x}_{-i}^T M\mathbf{x}_{-i} - 2x_i\mathbf{k}_1^T M\mathbf{x}_{-i} + \mathbf{k}_1^T\mathbf{k}_1 x_i^2 + k_2 x_i^2 \tag{6}$$

For (1) and (2) to be consistent, we need

$$\begin{aligned} M &= \Sigma_{N-1,N-1} + \Omega_{N-1,N-1} \\ -\mathbf{k}_1^T M \mathbf{x}_{-i} &= \sum_j \sigma_{ij} x_j \\ \mathbf{k}_1^T \mathbf{k}_1 + \mathbf{k}_2 &= -\alpha_i - \frac{1}{2} \end{aligned} \tag{7}$$

As (3) needs to hold for any $\mathbf{x}$, it must be

$$-k_1^T M = \tilde{\Sigma}_i^T$$

where

$$\tilde{\Sigma}_i = \begin{pmatrix} \sigma_{i1} \\ \sigma_{i2} \\ \dots \\ \sigma_{iN} \end{pmatrix}$$

Assume M is invertible

$$\mathbf{k}_1^T = -\tilde{\Sigma}_i^T M^{-1}$$

Thus

$$\mathbf{k}_2 = -(\alpha_i + \frac{1}{2} - \tilde{\Sigma}_i^T M^{-1} (M^T)^{-1} \tilde{\Sigma}_i)$$

A.3 Example: Teams of Size 3

In this problem, workers choose to enter the market for teams of size 3. Each worker then chooses their role within the (symmetric) team: position 1, position 2, or position 3. The social planner's maximization problem is as follows:

$$\int_0^1 \int_0^1 \int_0^1 M(x,y,z) \tilde{F}(x,y,z) dxdydz \tag{8}$$

$$\text{s.t.} \int_0^1 \int_0^1 M(x,y,z) dydz = 1 \ \& \int_0^1 \int_0^1 M(x,y,z) dxdz = 1 \ \& \int_0^1 \int_0^1 M(x,y,z) dxdy = 1$$

Apply Entropy-regulated Optimal Transport:

$$W(c) = \max \int_0^1 \int_0^1 \int_0^1 M(x,y,z) \tilde{F}(x,y,z) dxdydz - c \int_0^1 \int_0^1 \int_0^1 M(x,y,z) log M(x,y,z) dxdydz \tag{9}$$

With:

$$M(x,y,z) = \mu(x) \nu(y) \lambda(z) exp\left(\frac{\tilde{F}(x,y,z)}{c}\right) \tag{10}$$

Next we plug in the values here:

$$W(c) = \max \int_0^1 \int_0^1 M(x,y,z)\tilde{F}(x,y,z)dxdydz$$
$$-c \int_0^1 \int_0^1 \int_0^1 M(x,y,z) log\left[\mu(x)\nu(y)\lambda(z)exp\left(\frac{\tilde{F}(x,y,z)}{c}\right)\right] dxdydz$$

$$W(c) = \max \int_0^1 \int_0^1 M(x,y,z)\tilde{F}(x,y,z)dxdydz$$
$$-c \int_0^1 \int_0^1 \int_0^1 M(x,y,z) \left[log\mu(x) + log\nu(y) + log\lambda(z) + logexp\left(\frac{\tilde{F}(x,y,z)}{c}\right)\right] dxdydz$$

$$W(c) = -c \int_0^1 \int_0^1 \int_0^1 M(x,y,z) \left[log\mu(x) + log\nu(y) + log\lambda(z)\right] dxdydz$$

Using our market clearing conditions yields:

$$W(c) = -c\left(\int_0^1 log\mu(x)dx + \int_0^1 log\nu(y)dy + \int_0^1 log\lambda(z)dz\right) \tag{11}$$

We want to show that as $c \to 0$ the problem 10 becomes problem 8. How do we show this? We take the market clearing equations and plug in 10.

$$\mu(x) \int_0^1 \int_0^1 \nu(y)\lambda(z)exp\left(\frac{\tilde{F}(x,y,z)}{c}\right) dydz = 1$$
$$\nu(y) \int_0^1 \int_0^1 \mu(x)\lambda(z)exp\left(\frac{\tilde{F}(x,y,z)}{c}\right) dxdz = 1$$
$$\lambda(z) \int_0^1 \int_0^1 \mu(x)\nu(yexp\left(\frac{\tilde{F}(x,y,z)}{c}\right) dzdy = 1$$

Gives us:

$$ln\mu(x) = cln \int_0^1 \int_0^1 \nu(y)\lambda(z)exp\left(\frac{\tilde{F}(x,y,z)}{c}\right) dydz$$
$$= cln \int_0^1 \int_0^1 exp\left(\frac{\tilde{F}(x,y,z) - ln\nu(y) - ln\lambda(z)}{c}\right) dydz$$

As $c \to 0$, this becomes a *directed search problem* whereby we take the maximum, yielding:

$$\ln \mu(x) \to max_{y,z}\tilde{F}(x,y,z) - ln\nu(y) - ln\lambda(z)$$
$$\ln \nu(y) \to max_{x,z}\tilde{F}(x,y,z) - ln\mu(x) - ln\lambda(z)$$

$$\ln\lambda(z) \to max_{x,y}\tilde{F}(x,y,z) - ln\mu(x) - ln\nu(y)$$

Implies

$$\underbrace{u(x)}_{x \text{ best option/wage}} = max_{y,z}\tilde{F}(x,y,z) - \underbrace{v(y)}_{y \text{ wage}} - \underbrace{l(z)}_{z \text{ wage}}$$

A.4 MMT Theory

Recall γ which is the optimal measure–Pass (2014) indicates that the support of γ, $sprt(\gamma)$, is our main object of interest. This area of interest is defined as the smallest closed subset of $M_1 \times M_2 \times ...M_I$ of full mass, $\gamma(sprt(\gamma)) = 1$

We start with a discussion of differential geometry, using Pass (2014) notation. Let $T_x M_i$ denote the tangent space of M_i at x_i, the dual being the cotangent space. We're using these to take some differentials (JP: having some trouble with this concept).

The key concepts they adapt are the *non-degeneracy* and *twist* conditions:

A.4.1 Non-Degeneracy

At a point $(x_1, x_2) \in M_1 \times M_2$–assuming $D^2_{x_1,x_2}$ has full rank and c is non-degenerate for some (x_1, x_2). Then there is a neighborhood N of (x_1, x_2) such that for any optimal measure γ, $N \cap sprt(\gamma)$ is contained in an n-dimensional Lipschitz submanifold of the product space. They then extend this to multi-marginal.

A.4.2 Twist

Assume that c is semi-concave and that for each fixed x_1 the map:

$x_2 \to D_{x_1}c(x_1, x_2)$ is injective on the subset $\{x_2 : D_{x_1}c(x_1, x_2)\ exists\ \} \subset M_2$ where c is differentiable with respect to x_1