Online Appendix

Brand Reallocation, Concentration, and Growth

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Contents

A Overview

In this appendix, we discuss, in line with the text, greater detail on model extensions and the quantitative implications in Appendix B and the estimation in Appendix C.

B Model Extensions

In this section, we discuss some model extensions and provide a general framework for key takeaways from each extension. We hope these extensions can provide tools for those interested in the nature of competition in branding and advertisement.

Table B1 presents different counterfactual scenarios from shutting down brand reallocation with three different assumptions, discussed in greater detail in this section. First, we focus on two different types of competition, monopolistic competition (where leaders do not have pricing power) and Cournot competition (where leaders complete on quantity rather than price as in the main text). Then we focus on

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endogenous brand maturity, e.g., where firms choose advertisement expenditures to build their brand capital.

Table B1: Counterfactual on Shutting Down Reallocation — Model Extensions

	Baseline	Monopolistic Competition	Quantity Competition	Endogenous Maturity
Change in Leader Share (p.p)	-11.11	-12.32	-13.94	-10.23
Change in Growth rate (p.p)	-0.321	-0.450	-0.334	-0.283
Welfare (BGP, p.p)	-1.93	-2.13	-1.82	-1.61
Welfare (Transition, p.p)	-1.33	-1.78	-1.21	-1.13

Notes: Reallocation tax and entry subsidies, outcomes in counterfactual. Source: Author calculations.

We find that the counterfactual scenarios with shutting down brand reallocation look very similar to the baseline case. Shutting down brand reallocation is most costly when leaders engage in monopolistic competition, because the gains from trade only increase welfare in aggregate. The brand reallocation shut down is least costly in markets with endogenous maturity because firms can build their brands in response to being unable to purchase other brands. However, the qualitative results look quite similar to the main text. We now discuss how we proceed for each exercise in turn.

B.1 Cournot Competition and Monopolistic Competition

In this extension, we consider two alternative price setting mechanisms. In Cournot competition, we assume firms compete through quantity; in monopolisitic competition, we assume all firms take as given the group-level price index, and set a constant markup according to the exogenous substitution elasticity. Assumptions regarding pricing mechanisms leaves equilibrium characterization virtually unchanged, except for how the marginal value of products are calculated. Thus, we stress that while the quantitative magnitudes of results should be in line with a detailed understanding of the underlying market structure, the main qualitative results from the text hold. To make the extensions comparable to the baseline model, we re-calibrate the parameters to match the same moments as in the baseline model.

When prices are set through monopolistic competition, all brands have the same markup $\frac{\sigma}{\sigma-1}$. In this case, the marginal value of a product to the leader is the same as the average value. The only incentive of product reallocation when prices are set through monopolistic competition is that the buying firms have more efficiency operating the focal product, there is no protective incentive. The reallocation creates only benefit and no cost. This setup is very similar to the one considered in Akcigit et al. (2016).

When prices are set through Cournot competition, the leader's products have a higher markup than fringe products. In this case the perceived substitution elasticity for leaders is $\epsilon(s) = \left(\frac{1}{\sigma}(1-s)+s\right)^{-1}$. Similar to the derivation of marginal profit in price compition, by totally differentiating the static profit equations we reach the following marginal profits given quantity competition:

$$\Pi'(\phi) = \left(1+2s\frac{\sigma-1}{\sigma}\right)s'(\phi) = \left[\frac{s(\sigma-1)}{s(\sigma-1)+1}+1\right]\frac{s(1-s)}{\sigma\phi}. \tag{OA.1}$$

B.2 Endogenous Maturity

After firms create brands, the brand appeal may expand through either consumer diffusion or through endogenous advertisement. In our model, for simplification purposes, we focused on natural brand maturity through consumer diffusion. In reality, some brands receive more internal firm investment than others, while it may be harder to grow brands that are already large as they hit diminishing returns. In the model, the natural diffusion implicitly assumes these effects cancel out and focus on exogenous maturity. In this section, we explore the endogenous development of brand capital.

The main message from this development is that the core findings of the paper do not change as a result of endogenous maturity. Yet, we do think this framework can provide an important framework and insight for those applying data on advertising and marketing expenditures.

We suppose firms can invest in the maturity of its products. Specifically, we assume the maturity rate ι is a choice variable. A firm choosing ι incurs a flow cost of $\chi(\iota)$. Due to the existing flow equations only needing input on the overall ι, it does not require a significant modification of the model. The value functions are the only objects that require adjustments. We turn to the value functions here:

$$\begin{aligned}(\rho+g_t)u_t(\mathbf{x}) =& \underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\frac{1}{\phi_t}\pi(\phi_t)}_{\text{Operating Profit}} + \underbrace{\max_{\iota}\ \iota(\bar{\beta}-\beta)\frac{\partial u}{\partial \beta}(\mathbf{x}) - \chi(\iota)\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Maturity}} \\ &+ \underbrace{\max_{\theta}\lambda(\theta)\mathbb{E}_{\gamma'}\Omega_t(\mathbf{x}',\mathbf{x}) - \theta\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{u}_t(\mathbf{x}).\end{aligned} \tag{OA.2}$$

$$\begin{aligned}(\rho+g_t)v_t(\mathbf{x}) =& \underbrace{e^{z+\alpha+\beta+\gamma}\left(1+\phi_t\right)\Pi'(\phi_t)\phi_t}_{\text{Operating Profit}} + \underbrace{\max_{\iota}\ \iota(\bar{\beta}-\beta)\frac{\partial v}{\partial \beta}(\mathbf{x}) - \chi(\iota)\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Maturity}} \\ &+ \underbrace{\max_{\theta}\lambda(\theta)\mathbb{E}_{\gamma'}\left[-\Omega_t(\mathbf{x}',\mathbf{x})\right]^{+} - \theta\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}),\end{aligned} \tag{OA.3}$$

To make this extension comparable to the baseline results, we keep other parameters fixed as in the baseline model, except for ι. We specify $\chi(\iota) = \chi_0\iota^2$. We calibrate χ_0 such that the average growth rate of products match the baseline target. The results are presented in Table B1.

B.3 Oligopoly Structure

Suppose in addition to the one group leader, each product group has two multi-product firms. At the group level, there are three state variables, the quality gap between leaders and fringes: ϕ_1 and ϕ_2; and the growth rate of quality g, where $\phi_1 = \frac{Q_1}{Q_F}$ and $\phi_2 = \frac{Q_2}{Q_F}$ and $Q = Q_F + Q_1 + Q_2$. The static pricing equilibrium is summarized by two market shares (s_1, s_2) for the leaders that jointly solve:

$$s_1 = \frac{\phi_1\mu(s_1)^{1-\sigma}}{\phi_1\mu(s_1)^{1-\sigma}+\phi_2\mu(s_2)^{1-\sigma}+\bar{\mu}^{1-\sigma}}, \tag{OA.4}$$

$$s_2 = \frac{\phi_2\mu(s_2)^{1-\sigma}}{\phi_1\mu(s_1)^{1-\sigma}+\phi_2\mu(s_2)^{1-\sigma}+\bar{\mu}^{1-\sigma}}, \tag{OA.5}$$

where the markup is given by:

$$\mu(s) = \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1}. \tag{OA.6}$$

Denote the equilibrium market shares as $\Pi(\phi, \phi')$ if the focal leader's quality gap is ϕ and the other leader's quality gap is ϕ':

$$\Pi(\phi, \phi') = \frac{s(\phi, \phi')}{\sigma\left[1 - s(\phi, \phi')\right] + s(\phi, \phi')}, \tag{OA.7}$$

Similarly, the profit share that accrues in aggregate to fringe firms is

$$\pi(\phi_1, \phi_2) = \frac{1 - s(\phi_1, \phi_2) - s(\phi_1, \phi_1)}{\sigma}. \tag{OA.8}$$

Dynamic Innovation and Reallocation Problem. To characterize the incentives for innovation and reallocation, we first write out the value of heterogeneous brands to leaders and fringe firms. For notational simplicity, we denote the vector of brand characteristics as $\mathbf{x}_{it} = \left(\alpha_{ij(i,t)k}, \beta_0, \beta_{it}, \gamma_{ij(i,t)}\right)$. Consider a brand currently operated by a fringe firm, with characteristics $\mathbf{x}$. To the fringe firm, this brand has value $u_t(\mathbf{x})$ that solves the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned}(\rho + g_t) u_t(\mathbf{x}) = & \underbrace{e^{\beta+\gamma}\left(1 + \phi_1 + \phi_2\right) \pi(\phi_1, \phi_2)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\frac{\partial u}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma'} \Omega_t(\mathbf{x}', \mathbf{x}) - \theta \kappa_s^{FL} \frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{u}_t(\mathbf{x}).\end{aligned} \tag{OA.9}$$

For a brand with state $\mathbf{x}$ that is currently operated by the group leader, its discounted value to the leader is

$$\begin{aligned}(\rho + g_t) v_t(\mathbf{x}) = & \underbrace{e^{z+\alpha+\beta+\gamma}\left(1 + \phi_1 + \phi_2\right) \Pi_1(\phi_1, \phi_2)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\frac{\partial v}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma'} \left[-\Omega_t(\mathbf{x}', \mathbf{x})\right]^{+} - \theta \kappa_s^{LF} \frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}),\end{aligned} \tag{OA.10}$$

$$\begin{aligned}(\rho + g_t) y_t^F(\mathbf{x}) = & \underbrace{e^{z+\alpha+\beta+\gamma}\left(1 + \phi_1 + \phi_2\right) \phi_1 \Pi_1(\phi_1, \phi_2)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\frac{\partial v}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma'} \left[-\Omega_t(\mathbf{x}', \mathbf{x})\right]^{+} - \theta \kappa_s^{LF} \frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}),\end{aligned} \tag{OA.11}$$

$$\begin{aligned}(\rho + g_t) y_t^L(\mathbf{x}) = & \underbrace{e^{z+\alpha+\beta+\gamma}\left(1 + \phi_1 + \phi_2\right) \Pi_2(\phi_1, \phi_2)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\frac{\partial v}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma'} \left[-\Omega_t(\mathbf{x}', \mathbf{x})\right]^{+} - \theta \kappa_s^{LF} \frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}),\end{aligned} \tag{OA.12}$$

C Estimation

In this section, we overview in greater detail the estimation process.

C.1 Algorithm to Solve Balanced Growth Path

1. Set $(t) = 0$ and guess $\frac{\mathbf{w}}{\mathbf{C}}^{(0)}$.
2. Given $\frac{\mathbf{w}}{\mathbf{C}}^{(t)}$, for each product group k, look for the group-level $\left(g_k^{(t)}, \phi_k^{(t)}\right)$ using the following sub-routines:

   a. guess $\phi_k^{(t)}$, solve for $v_k(\mathbf{x})$, $x_k(\mathbf{x})$, $u_k(\mathbf{x})$, and $\Omega_k(\mathbf{x}', \mathbf{x})$ by solving equations (OA.11), (19), and (OA.9) and corresponding policy functions, setting the time derivative to zeros;

   b. Given the policy functions from step (a), solve for the equilibrium density $n_k^L(\mathbf{x})$ and $n_k^F(\mathbf{x})$ using equation (17) and (16), by setting time derivatives to zeros;

   c. Look for the equilibrium (g_k, ϕ_k) such that the free-entry condition (21) and the definition of ϕ hold. This step could be performed easily by any root finding routines.

3. With the equilibrium objects from step 2, aggregate to aggregate labor inputs, by Proposition (3);
4. Update $\frac{\mathbf{w}}{\mathbf{C}}^{(t+1)}$ according to the labor supply condition 32;
5. Iterate until $\frac{\mathbf{w}}{\mathbf{C}}^{(t)}$ and $\frac{\mathbf{w}}{\mathbf{C}}^{(t+1)}$ are sufficiently close to each other.

C.2 Algorithm to Solve Transitional Path

To solve for the transitional path, we start from two sets of BGP, which can be solved as in Section C.1.

1. Set $iter = 0$. Guess a time path of wage-GDP ratio $\frac{\mathbf{w_t}}{\mathbf{C_t}}^{(iter)}$. We set the time length to be 100 years.

2. Given $\frac{\mathbf{w_t}}{\mathbf{C_t}}^{(iter)}$, for each product group k, look for the group-level $\left(g_{kt}^{(t)}, \phi_{kt}^{(t)}\right)$ using the following sub-routines:

   a. guess ϕ_{kt}, solve for $v_{kt}(\mathbf{x})$, $x_{kt}(\mathbf{x})$, $u_{kt}(\mathbf{x})$, and $\Omega_{kt}(\mathbf{x}', \mathbf{x})$ by solving equations (OA.11), (19), and (OA.9) and corresponding policy functions, setting the last period values to be the new BGP values of value functions;

   b. Given the policy functions from step (a), solve for the equilibrium density $n_{kt}^L(\mathbf{x})$ and $n_{kt}^F(\mathbf{x})$ using equations (16) and (17), by setting the initial value of density to be the old BGP values;

   c. Look for the equilibrium (g_{kt}, ϕ_{kt}) such that the free-entry condition in equation (21) and the definition of ϕ hold. In practice, iteration on the time paths proves to work well.

3. With the equilibrium objects from step 2, aggregate to aggregate labor inputs, by Proposition (3);
4. Update $\frac{\mathbf{w_t}}{\mathbf{C_t}}^{(iter+1)}$ according to the labor supply condition in equation (32);
5. Iterate until $\frac{\mathbf{w_t}}{\mathbf{C_t}}^{(iter)}$ and $\frac{\mathbf{w_t}}{\mathbf{C_t}}^{(iter+1)}$ are sufficiently close to each other.

References

Akcigit, U., M. A. Celik, and J. Greenwood: 2016, 'Buy, Keep, or Sell: Economic Growth and the Market for Ideas'. *Econometrica* **84**(3), 943–984.